Modelling multiple networks

Outline

Why multiple networks?

Analytic possibilities

- **Blockmodels** for multiple networks
- **QAP**: correlations among networks
- **Exponential random graph models** for multiple networks
  - Homogeneity assumptions
  - Estimation

Examples

- Collaboration, Advice and Friendship in a Law Firm (Lazega & Pattison, 1999; Lazega, 2000)
- Cognitive social structures (Koehly & Pattison, 2005)
- Group cohesion (Albert, 2002)

Why multiple networks?

There is rarely one network!

One network provides a context for another: there are likely to be interdependencies ***across*** networks, eg

- Acquaintance networks may provide opportunities (as well as constraints) for close friendship networks
- Business partners may be selected from friends' acquaintances (information, trust and reputation at work)

We find that a particular kind of tie in a particular kind of context tends to have a particular type of "structural logic" or "signature". Overlap in networks (***multiplexity***) may lead to the logic of one form of tie being transferred to another: how, and what is the consequence?

- friends in the workplace may affect styles of work interaction
- nepotism, cronyism

In general, the interdependencies between networks of different types may extend well beyond the dyad: hence we need methods for understanding the way in which ties of different types interlock and influence one another

Some analytic possibilities

Analysis of positions and roles across multiple networks

- Blockmodel algorithms (CONCOR etc)
- Stochastic blockmodels (BLOCKS)

Relationship at the dyadic level: statistical evaluation

- QAP

In search of extra-dyadic interdependencies

- Interdependencies among labelled paths in multiple networks, in order to characterise the patterns of interlocking of different types of tie (Boorman & White, 1976; Pattison,1993)
- Statistical evaluation of path interdependencies in multiple networks using conditional random graph distributions (Pattison et al, 2001; PACNET in Stocnet)
- Exponential random graph models (Pattison & Wasserman, 1999; Koehly & Pattison, 2005) with and without relational "covariates"

Visualisation

- Needs much work

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Exponential random graph models for multiple networks in three steps

1. Instead of tie variables $\mathbf{X} = [X_{ij}]$ with *realisations* $\mathbf{x} = [x_{ij}]$, we have ***multi-relational tie variables*** $\mathbf{X} = [X_{ijm}]$ with *realisations* $\mathbf{x} = [x_{ijm}]$, where X_{ijk} refers to the tie from actor i to actor j ***on relation m***

2. Exactly the same general theory for interactive variable systems holds

3. Homogeneity assumptions: equate parameters for configurations with the same shape ***and the same relation labels***

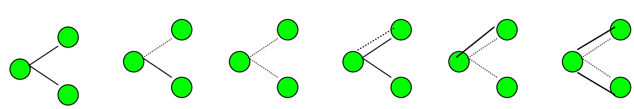
Parameters for multiple network exponential random graph models

Suppose we have two relations A,B:

Edges

A B AB

Two-stars



A,A A,B B,B AB,A AB,B AB,AB

etc…

Example 1: Lazega (2000)

Lazega's law firm:

71 lawyers: 36 partners, 35 associates

We model two networks:

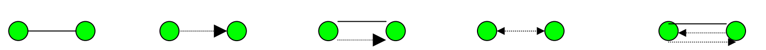
Collaboration (as before)

Advice-seeking

We use Markov models/pseudo-likelihood estimation – still working on programming new specifications/MCMCMLE for multiple networks…

Note: Analysis using collaboration, advice-seeking *and* friendship in reading pack (Lazega & Pattison, 1999)

Collaboration and advice model
1. edge and dyad parameters



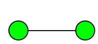
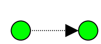
-3.37 (.22)	-2.93 (.21)	2.60 (.20)	1.27 (.17)	-.40 (.11)

Both networks are relatively sparse

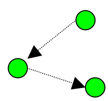
Advice and work ties tend to be aligned

Advice ties tend to be reciprocated

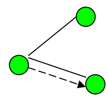
But multiplexity and reciprocation tend to involve different advice ties

Key: *Work*  *Advice* 

Collaboration and advice model
2. 2-star parameters



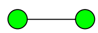
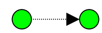
-.08 (.01)



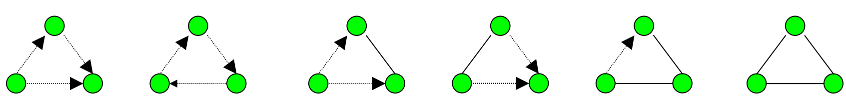
-.04(.01)

Advice "holes" are relatively rare

Advice and collaboration ties tend not to coincide for those with many collaborators

Key: *Work*  *Advice* 

Collaboration and advice model
3. triangle parameters



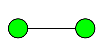
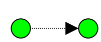
.30 (.01) -.23 (.03) -.06 (.01) -.05 (.01) .14 (.02) .11 (.01)

Advice ties are transitive and acyclic: hierarchical (with some clustering)

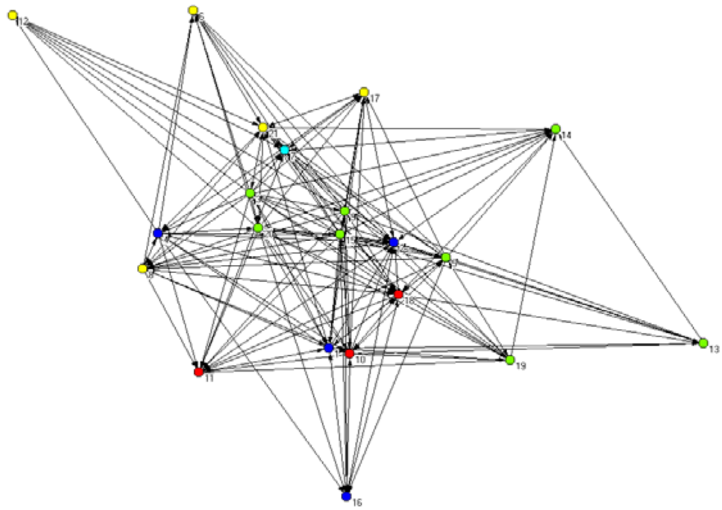
Collaboration ties are clustered

Common advisors tend not to be collaborators and collaborators tend not to have common advisors

Advice ties tend to span collaboration paths (checking up on a collaborator, collaboration opportunity through advice, or advice-based brokering for collaborators?)

Key: *Work*  *Advice* 

Example 2. Advice network for managers in a high-tech firm (Krackhardt, 1987): self-reported ties



Example 2: Models for *cognitive social structures* (Krackhardt, 1987; Koehly & Pattison, 2005)

Each of the 21 managers was actually asked:

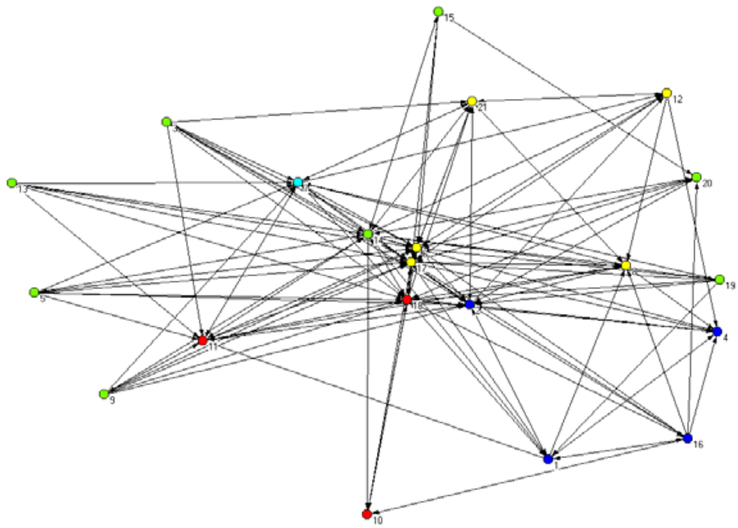
"*Who would [i] go to for advice at work*?" for each of the 21 possible *i*'s

Relational observations

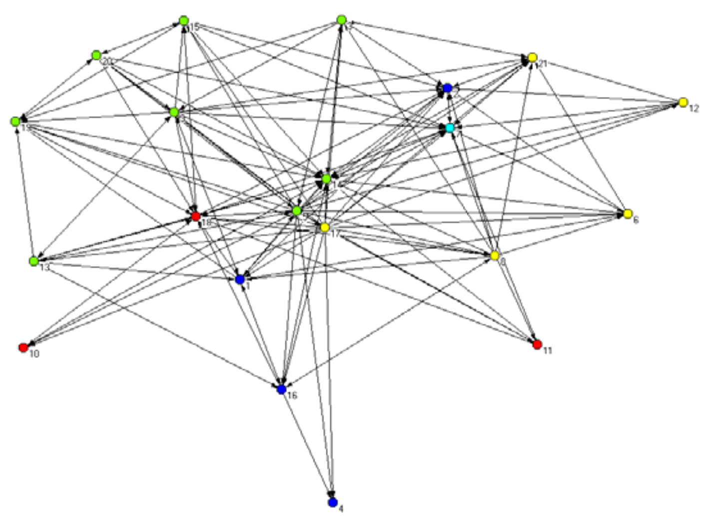
X_{ijm} = 1 if *m* reports that *i* seeks advice from *j*
0 otherwise

We model Pr (**X**=**x**), setting parameters for relational configurations to be equal if they are structurally identical from *m*'s perspective apart from a relabelling (we distinguish the ***source***, ***target*** and ***third party observer*** in the report of any tie)

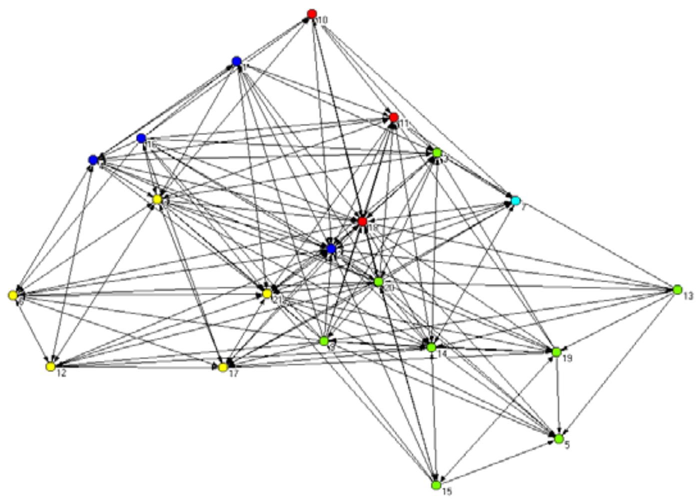
CEO's perspective
(CEO=7; colour indicates department)



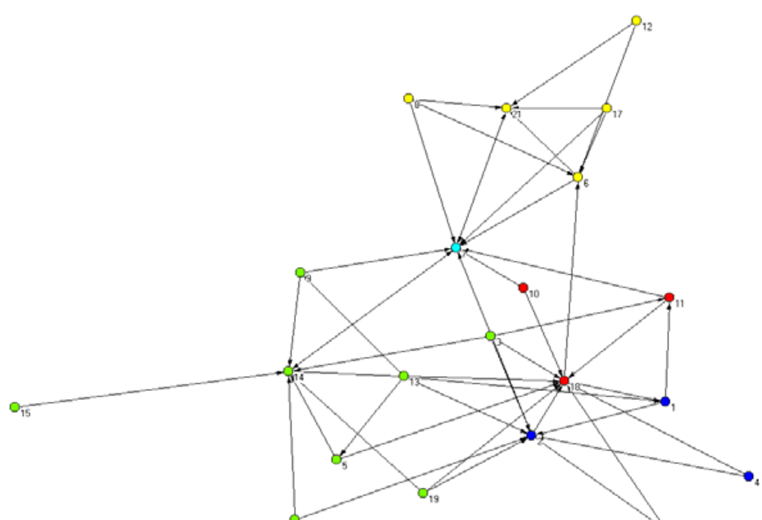
A vice-president's perspective
(node 14)



Another vice-president's perspective (node 21)



Perspective of the newest manager (node 13)



i. Bernoulli model (a): colours code (relative) identity of actors & perceivers

Perception of advice ties by third parties

-0.89 (0.02)

Perception of outgoing advice ties

-0.19 (.09)

Perception of incoming advice ties

0.05 (.10)

ii. Bernoulli model (b): consensus effects

consensus between:

-2.60 (.05)

distinct third parties 0.24 (.01)

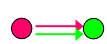
-1.45 (.17)

source & third party 0.18 (.03)

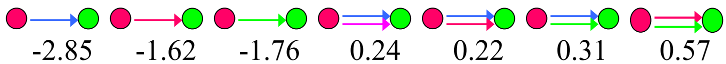
-1.52 (.18)

target & third party 0.27 (.03)

source & target 0.49 (.17)



iii. A dyad-independent model: consensus and reciprocity



reciprocity effects:

a third party 1.49

distinct third parties -0.04

third party & target -0.06

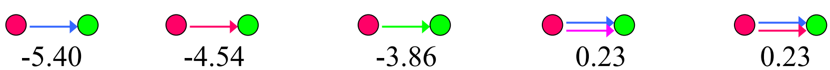
third party & source -0.09

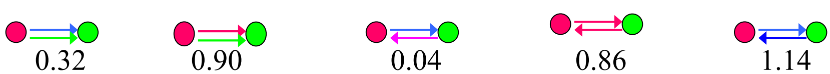
target & source -0.32

target & source (reciprocal) -0.30

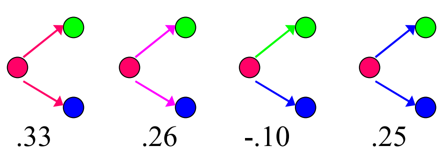
target or source 1.11

iv. A restricted Markov model: consistency and bias





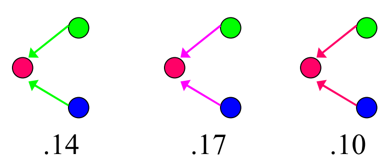
differentiation in advice-seeking:



perceived by:

source, third party, target & third party, a target

differentiation in being sought:

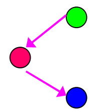
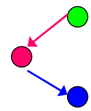
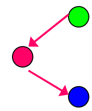


perceived by:

a source, third party, target

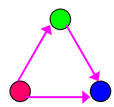
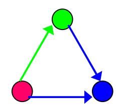
Consistency and bias, *continued*

advice-seeking paths:

	-.12	-.13	-.09
perceived by:	third party	targets	sources

consistency and bias

	.10	.17
perceived by:	third party	targets

Cognitive social structures

ties involving the perceiver are more likely
perceivers report more ties as targets; tend to be confirmed
consensus effect for target and source
third party consensus effects
reciprocity is *perceived* by sources, targets and third parties
no independent effect of what normally counts as reciprocity()
sources, targets and third parties perceive individual differences in advice-seeking patterns (not necessarily the same ones!)
weaker *perceptual* effects for advisors
third parties perceive transitivity
transitivity effect for advisors (who comes to you for advice?)

Perceptions reflect a shared reality, but not a simply measured one. Structural effects are subject to local consensus. The social world is seen as ordered "at a distance" and sometimes more ordered than it is perceived to be by the immediate players. In this case, advisors report more consensual effects than advice-seekers.

Example 3: An office of a management consulting firm (n = 37; Albert, 2002)

Albert (2002): functional, structural and cognitive interdependencies among office members (after Lindenberg, 1997):

functional interdependence (treated here as exogenous)

shared work groups $G_{ij} = 1$ if i, j in same group

$G_{ij} = 0$ otherwise

structural interdependence

"Of all the people you work with, who do you talk to when you want to find out information at Alpha?" $X_{ij} = 1$ if i nominates j

$X_{ij} = 0$ otherwise

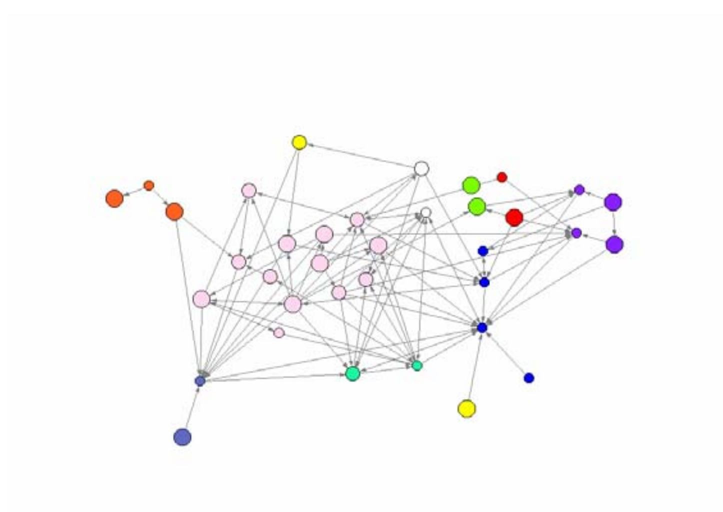
cognitive interdependence

e.g., "Our work group is united in trying to reach its goals for performance"

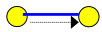
$Y_{ik} = 1$ if i endorses item k

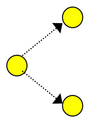
$Y_{ik} = 0$ otherwise

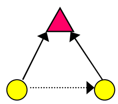
Information network in a consulting firm: colour codes workgroup, size codes beliefs

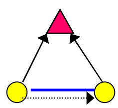


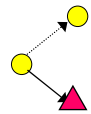
Markov model for Pr(X=x,Y=y | G=g)
Note: shared workgroup treated as exogenous

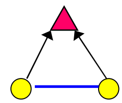

3.01 (.91)

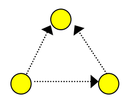

0.19 (.05)


-0.50 (.27)


1.94 (.56)


-0.75 (.22)


0.72 (.28)

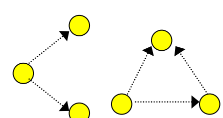

0.34 (.10)

Key: actor, item
information tie
endorsement of item
shared group

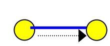
Functional, structural and cognitive interdependence

Separable tendencies for:

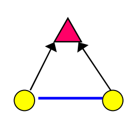
structural logic of information seeking: hierarchical with differentiation in information seeking — *structural interdependence*



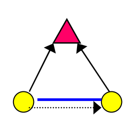
information ties within groups — *structural & functional interdependence*



shared beliefs within groups — *cognitive and functional interdependence*



shared beliefs within groups among those linked by an information tie — *cognitive, structural and functional interdependence:*



This is group cohesion: is it also what we really mean by social capital?

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