Algorithms for network positions

Outline

Measuring structural equivalence

- Distance-based similarity measure
- Correlation measure

Algorithms for approximate structural equivalence

- CONCOR
- STRUCTURE
- Tabu Search, and other combinatorial optimisation approaches

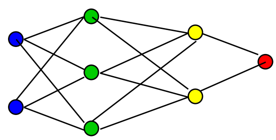
Some practical issues

- Including the transpose?
- How many blocks?
- Which algorithm?

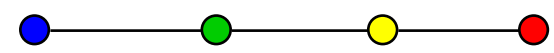
Algorithms for approximate regular equivalence

Structural equivalence

Nodes *a* and *b* are ***structurally equivalent*** if, whenever (*a*,*x*) is an edge in *G* then so is (*b*,*x*), and conversely



graph



blockmodel

00111000
00111000
11000110
11000110
11000110
00111001
00111001
00000110

adjacency matrix for graph

0100
1010
0101
0010

adjacency matrix for blockmodel

Measuring structural equivalence

Two nodes are ***structurally equivalent*** if and only if the corresponding rows (or columns) of the adjacency matrix are identical

We can therefore measure the ***degree*** to which two nodes are structurally equivalent by measuring the degree to which their columns are identical:

A distance measure

e.g., square root of sum of squared discrepancies between corresponding column entries (Euclidean distance): Burt's **STRUCTURE** program

A proximity measure

e.g., correlation coefficient between columns (Pearson or phi coefficient): Breiger, Boorman & Arabie's (1975) **CONCOR** algorithm

Directed graphs:

the **transpose** of the adjacency matrix is usually stacked below the matrix

to measure similarity across both rows and columns of the original matrix

Multiple graphs:

the matrices are usually stacked one below another

to measure similarity across all graphs

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Early blockmodelling algorithms

Clustering algorithms can then be used to determine blocks of approximately structurally equivalent nodes

Algorithms may be:

- ***Hierarchical*** or ***non-hierarchical***
- ***Divisive*** or ***agglomerative***

Early algorithms:

- **STRUCTURE** (hierarchical, agglomerative)
- **CONCOR** (hierarchical, divisive)

More recent algorithms

Define a criterion of goodness of fit of a partition to an equivalence form

Use a combinatorial optimisation algorithm to find a partition with optimal fit

Examples:

- **UCINET: Tabu search**
- **Pajek: local optimisation**
  - start with some partition, and replace the current partition with any partition in its “neighbourhood” that has a more optimal fit; repeat until no further improvement is possible

Doreian data in DL format

```
DL n=14 format=fullmatrix
labels
"Executive"
"Auditor"
"Sheriff"
"Council 1"
"Council 2"
"Council 3"
"Council 4"
"President"
"Council 5"
"Council 6"
"Fr Council"
"Fr Pres"
"City Mayor"
"Prosecutor"
data:
0 0 1 1 0 0 1 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1 1 1 0 1 0 0
1 0 0 1 0 1 1 0 0 0 0 0 0 1
1 0 1 0 1 1 1 0 0 0 0 1 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0
0 0 1 1 1 0 0 0 0 0 0 1 0 1
1 0 1 1 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 1 1 0 1 1 0
0 1 0 0 0 0 0 1 0 1 0 0 1 0
0 1 0 0 0 0 0 1 1 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 1 0 1 0 1 0 1 0 0 1 0 1 0
1 0 0 0 0 0 0 1 1 1 0 1 0 0
0 0 1 0 0 1 0 0 0 0 0 0 0 0
```


UCINET 6: CONCOR

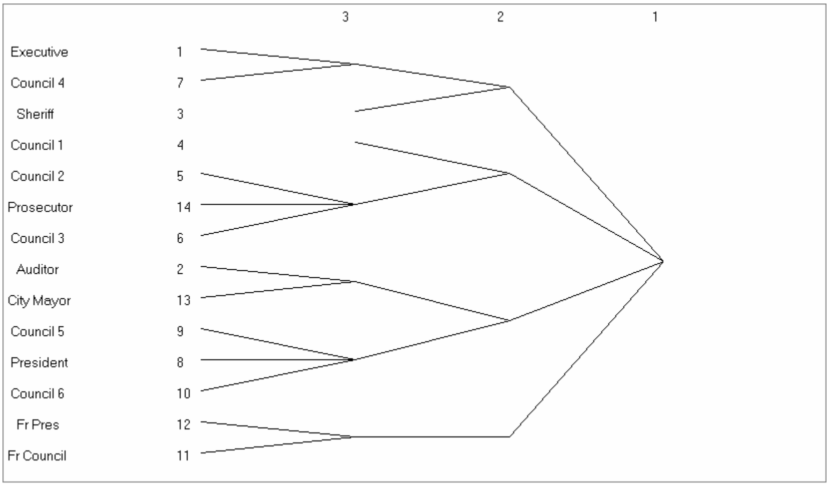
In UCINET 6, read in data in DL format and then select:

Network
    Roles & Positions
        Structural
            CONCOR

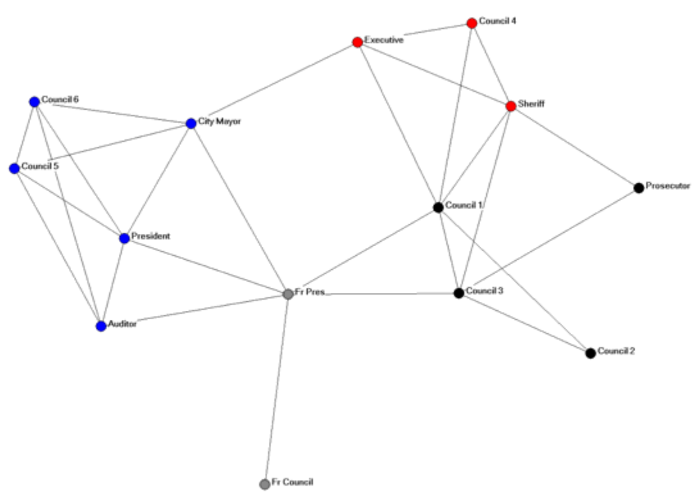
[choose max depth of splits = 3,
defaults, otherwise]

Use "ConcorCCPart" as attribute for colouring nodes

CONCOR partition



4-block CONCOR solution



Selected output

Blocked Matrix

```
                           1           1   1   1 1
                 1 3 7   5 4 6 4   2 9 0 8 3   2 1
                 E S C   C P C C   A C C P C   F F
                -------------------------------------
  1  Executive |   1 1 |       1 |         1 |     |
  3    Sheriff | 1   1 |   1 1 1 |           |     |
  7  Council 4 | 1 1   |       1 |           |     |
               ---------------------------------------
  5  Council 2 |       |     1 1 |           |     |
 14 Prosecutor |   1   |     1   |           |     |
  6  Council 3 |   1   | 1 1   1 |           | 1   |
  4  Council 1 | 1 1 1 | 1   1   |           | 1   |
               ---------------------------------------
  2    Auditor |       |         |   1 1 1   | 1   |
  9  Council 5 |       |         | 1   1 1 1 |     |
 10  Council 6 |       |         | 1 1   1 1 |     |
  8  President |       |         | 1 1 1   1 | 1   |
 13 City Mayor | 1     |         |   1 1 1   | 1   |
               ---------------------------------------
 12    Fr Pres |       |     1 1 | 1     1 1 |   1 |
 11 Fr Council |       |         |           | 1   |
                --------------------------------------
```

Density Matrix

```
      1     2     3     4
   ----- ----- ----- -----
1  1.000 0.417 0.067 0.000
2  0.417 0.667 0.000 0.250
3  0.067 0.000 0.900 0.300
4  0.000 0.250 0.300 1.000

R-squared = 0.500
```


A 4-block solution using “optimisation”

```
Results

Number of errors: 24
R-square = 0.520

Errors per block

     1 2 3 4
     - - - -
  1  2 1 0 3
  2  1 6 2 1
  3  0 2 0 1
  4  3 1 1 0


Block Assignments:

    1:  Auditor President Council 5 Council 6 City Mayor
    2:  Executive Council 2 Council 3 Council 4 Prosecutor
    3:  Sheriff Council 1
    4:  Fr Council Fr Pres
```

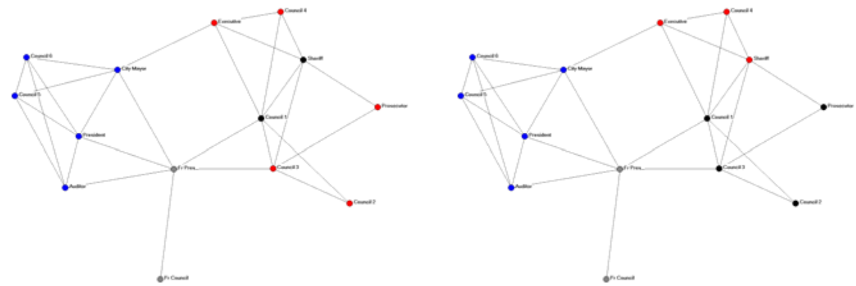
Blocked Adjacency Matrix

```
                     1 1             1           1 1
                 8 2 0 3 9   7 5 6 1 4   4 3   2 1
                 P A C C C   C C C E P   C S   F F
                -------------------------------------
  8  President |   1 1 1 1 |           |     | 1   |
  2    Auditor | 1   1   1 |           |     | 1   |
 10  Council 6 | 1 1   1 1 |           |     |     |
 13 City Mayor | 1   1   1 |       1   |     | 1   |
  9  Council 5 | 1 1 1 1   |           |     |     |
               ---------------------------------------
  7  Council 4 |           |       1   | 1 1 |     |
  5  Council 2 |           |     1     | 1   |     |
  6  Council 3 |           |   1     1 | 1 1 | 1   |
  1  Executive |       1   | 1         | 1 1 |     |
 14 Prosecutor |           |     1     |   1 |     |
               ---------------------------------------
  4  Council 1 |           | 1 1 1 1   |   1 | 1   |
  3    Sheriff |           | 1   1 1 1 | 1   |     |
               ---------------------------------------
 12    Fr Pres | 1 1   1   |     1     | 1   |   1 |
 11 Fr Council |           |           |     | 1   |
                --------------------------------------
```

Density Table

```
      1    2    3    4
   ---- ---- ---- ----
1  0.90 0.04 0.00 0.30
2  0.04 0.30 0.80 0.10
3  0.00 0.80 1.00 0.25
4  0.30 0.10 0.25 1.00
```

Comparing CONCOR (left) and optimisation (right) solutions



Some practical issues

Include the transpose for directed graphs?

Probably, provided that there are no missing respondents

How many blocks?

- Tricky question
- Purpose is important:
  - many blocks can be good for producing visualisations
  - few blocks can be more useful if major variations in position of primary interest
- Heuristics from cluster analysis
- Embed within a statistical framework for a more principled approach

Which algorithm?

- CONCOR works remarkably well as a quick, exploratory tool
- Optimisation is valuable in less exploratory contexts
- A statistical framework can be exceptionally helpful

Regular equivalence on UCINET

Select:

Network
    Roles & Positions
        Maximal Regular
            Optimization

And then specify Input Dataset and Number of blocks (3, here)

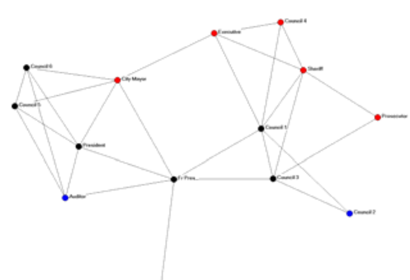
Regular Equivalence via Optimization

```
•    REGULAR BLOCKMODELS VIA TABU SEARCH
•    -----------------------------------------------------
     --------------------------

•    RESULTS:
•      No. of errors: 0.000

•    Blocked Adjacency Matrix

•                           1     1       1 1         1
•                         1 4 3 7 3   9 4 2 0 8 6   2 5 1
•                         E P S C C   C C F C P C   A C F
•                        ---------------------------------
•      1  Executive |     1 1 1 |   1         |       |
•     14 Prosecutor |     1     |           1 |       |
•      3    Sheriff | 1 1   1   |   1       1 |       |
•      7  Council 4 | 1   1     |   1         |       |
•     13 City Mayor | 1         | 1   1 1 1   |       |
•                   -----------------------------------
•      9  Council 5 |         1 |       1 1   | 1     |
•      4  Council 1 | 1   1 1   |     1     1 |   1   |
•     12    Fr Pres |         1 |   1     1 1 | 1   1 |
•     10  Council 6 |         1 | 1       1   | 1     |
•      8  President |         1 | 1   1 1     | 1     |
•      6  Council 3 |   1 1     |   1 1       |   1   |
•                   -----------------------------------
•      2    Auditor |           | 1   1 1 1   |       |
•      5  Council 2 |           |   1       1 |       |
•     11 Fr Council |           |     1       |       |
•                    ----------------------------------
```



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