Neighborhood-based models for social networks[#]

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Abstract

We argue that social networks can be modeled as the outcome of processes that occur in overlapping local regions of the network, termed *local social neighborhoods*. Each neighborhood is conceived as a possible site of interaction and corresponds to a subset of possible network ties. In this paper, we discuss hypotheses about the form of these neighborhoods, and we present two new and theoretically plausible ways in which neighborhood-based models for networks can be constructed. In the first, we introduce the notion of *a setting structure*, a directly hypothesized (or observed) set of exogenous constraints on possible neighborhood forms. In the second, we propose higher-order neighborhoods that are generated, in part, by the outcome of interactive network processes themselves. Applications of both approaches to model construction are presented, and the developments are considered within a general conceptual framework of locale for social networks. We show how assumptions about neighborhoods can be cast within a hierarchy of increasingly complex models; these models represent a progressively greater capacity for network processes to "reach" across a network through long cycles or semi-paths. We argue that this class of models holds new promise for the development of empirically plausible models for networks and network-based processes.

1. Introduction

The importance of network structure for a wide variety of social processes has been convincingly demonstrated in many different domains, including interpersonal influence (Friedkin, 1998), the spread of disease (Klovdahl, 1985; Kretzschmar & Morris, 1996), and information diffusion (Valente, 1995). Yet the success of attempts to build adequate models for network-dependent processes is critically dependent on our understanding of the *structure* of social networks and particularly on our capacity to construct adequate models for network structure. Frank and Strauss (1986) took a significant step in addressing the problem of model construction when they introduced the class of Markov random graphs. Within this class of models, it is possible to relinquish the statistically convenient but empirically implausible assumption of the independence of network ties in different dyads (that is, of ties linking non-identical *pairs* of people). Relaxation of the dyad-independence assumption has led to the construction of new and demonstrably more successful classes of random graph models for networks (Pattison & Wasserman, 1999; Skvoretz & Faust, 1999; Robins, Pattison & Wasserman, 1999; Wasserman & Pattison, 1996). In this paper, we propose a more general theoretical framework for building models of network structure, and we show how the approach leads to a hierarchy of models within which hypotheses about the nature of network structure can be explored.

Like many who have previously attempted to understand network structure, we begin with the premise that network structures are, at least in part, generated "locally". We see the major challenge facing model-builders as that of understanding the precise form and extent of appropriate local dependencies. Our approach leads to a generalisation of a number of well-known classes of statistical models for network structures and is, at the same time, sympathetic to some important theoretical claims about the localised nature of important network processes (e.g., Abbott, 1997; Granovetter, 1973; Johnsen, 1986).

Before articulating some more general ways in which the term "local" might be understood, we note that a number of significant contributions to the social networks literature have demonstrated how certain systematicities in local *triadic* social network structures can give rise to particular global network properties. Prominent examples include balance theory and its generalizations (e.g., Cartwright & Harary, 1978; Davis, 1967), Granovetter's (1973, 1982) strength-of-weak-ties argument and Johnsen's (1986) analysis of microstructures underlying the formation of friendship networks. In each of these examples, systematic features of local network structure that are specifiable at the level of small network substructures – triples of individuals and the ties that connect them – are seen to have potentially profound implications for aspects of global network structure. Below, we argue that such claims can be represented using models in which network ties are modeled as random variables that are subject to various kinds of regular, "locally specified" constraints.

The insistence on a stochastic formulation as well as a local one may seem surprising, but work by Watts on the Small World Phenomenon (Watts & Strogatz, 1998; Watts, 1999a, b) has demonstrated how the introduction of even a small proportion of random network ties to a "regular" network structure can have a dramatic effect on the connectivity properties of a network. In addition, considerations of scale encourage a stochastic approach. As White (1992) has argued, "no larger ordering which is deterministic either in cultural assertion or social arrangement could sustain and reproduce itself across so many and such large network populations as in the current world. Some sort of stochastic environment must be assumed and requires modeling" (pp. 164-165).

Beyond triads

Despite the well-established claims for the importance of regularities in triadic network configurations, few approaches to network modeling have considered *extra-triadic* features as the basis for building models for global structures. Rather, higher order local configurations – for instance, those involving four or more actors – are assumed to arise as the concatenation of lower-order dyadic and triadic configurations. It is not clear, however, that this assumption can always be justified, either theoretically or empirically. For instance, theoretical arguments about generalized exchange (e.g., Bearman, 1997) suggest the importance of cycles involving more than three actors. In addition, we may not be able to explain the presence of long paths in a network as accidental conjunctions of shorter paths involving no more than three actors. For example, people may use their friends to get in touch with other individuals with certain qualities. If those qualities are individual attributes, then this process can be represented as a path involving three people. But if the quality is a network-related property – for instance, being "well-connected" or being connected in a particular way – then more than three actors are implicated, even though the process is local. Indeed, such a network process may itself creates new boundaries for "locality" as it unfolds.

So, in order to explore the importance of longer cycles and paths in networks, we need to construct models that incorporate local structures involving more than three actors. And as longer cycles and paths may be crucial to certain global properties of the network (e.g. connectivity), it is apparent that we need theoretical conceptualizations and empirical methods that expand the possibility of what counts as local. We argue below that what is "local" in a network needs to be conceptualised carefully, and may even need to incorporate features that are exogenous to a network. To the extent that this claim is justified for certain types of networks, it has implications not only for the models that we might need to construct but also for the type of observations that we might usefully make in network studies.

In what follows, then, we consider hybrid definitions of "locality" that comprise triadic, or other types of, dependencies constrained by exogenous features. And we introduce methods to model notions of "locality" that emerge from the network processes themselves, thereby allowing us to investigate claims about longer cycles and paths. We base these innovations on a theoretical consideration of what we term *social locales* and *social neighborhoods*.

Social locales and neighborhoods

Our local approach to model building recognizes the socially situated nature of social action, the fact that social action takes place within *social locales* (e.g., Abbott, 1997; Feld, 1981; White, 1992). But what do we mean by the term "social locale"? Is a locale defined by geographical location, a set of network ties, or some aspect of community or culture? Like White (1992), we argue that all of these are likely to be implicated in the notion of locale. We do not attempt a precise definition here, but we propose that locale be regarded as a complex relational entity that links the geographical, social, cultural and psychological aspects of the context for social action.[1] The importance of social locale in developing models for network structure is that the (often unobserved) characteristic of context implicit in the notion of locale is the generator of contingencies among possible network ties.

Consider, for example, the so-called forbidden triad in Granovetter's (1973) theory. The triad comprises a triple of individuals in which two pairs of individuals are each linked by

[1] Feld (1981) introduced a related notion of *focus*, "a social, psychological, legal, or physical entity around which joint activities are organized" (p.1016), and he argued that foci underpin aspects of patterning in social networks.

a strong tie, and the remaining pair by neither a strong nor weak tie. Granovetter's proposition that such triads are scarce can be seen as a claim that an individual with strong ties to two others is unlikely to be able to maintain these dyadic relations as components of separate locales. Rather, a locale encompassing all three individuals is likely to emerge (through, for example, representations in discursive forms, or common physical settings) and the locale is likely to have implications for the potential ties that it contains. The original dyadic locales may persist, but they are likely to be augmented by higher-order triadic locales. More generally, we see locales as overlapping and as operating at different "scales", from possibly more intimate dyadic contexts to broader and progressively more public and communal social settings.

We argue that previous approaches to the development of network models have made implicit assumptions about the nature of locale. At the simplest level, locales have been regarded as entities that correspond to single possible network ties, instantiating a class of (Bernoulli) models in which each tie is assumed to be negotiated independently of each other tie (Frank, 1981). The general implausibility of the claim that the tie from one individual to another is negotiated independently of the tie from the second to the first led to the somewhat more general notion of locale as *dyad*. In this formulation, a locale corresponds to a pair of individuals and the possible interdependent ties between them, but network processes occurring within separate dyadic locales are assumed to be independent of one another. This assumption underpins the so-called p_1, or dyad-independent, class of models (Holland & Leinhardt, 1981; Wasserman, 1987; Wasserman & Galaskiewicz, 1984). The notion that locales are restricted to dyads has also been criticised and Frank and Strauss made a significant generalisation with the introduction of Markov random graphs. They proposed that a pair of possible network ties that share *one or more* actors belong to a common locale, and so introduced a model for network structure in which locales are potentially overlapping. The overlapping nature of locales means that the outcome of processes in one locale may have some impact on processes within another locale, thus lending a self-organising quality to the resulting characterisation of network structure.

The characterization just presented is based on two implicit assumptions. First, it is assumed that network ties are subject to both local and random processes. Second, it is assumed that systematicities in the arrangement of network ties emerge from regular, interactive processes occurring within social locales. Each locale is associated with a subset of possible network ties that we term the *local social neighborhood* of the locale. The neighborhood of the locale comprises its potential interpersonal relational components, with the geographical, psychological and sociocultural aspects stripped away. The significance of this construct for building network models is that network data are often observed in isolation from these other contextual aspects, and the neighborhood may be the only aspect of a locale that is available for modeling. We note that this sense of social neighborhood is distinct from existing network concepts such as cliques or blocks. Cliques and blocks are defined in terms of observed ties in a network, whereas a social neighborhood specifies interdependencies among *possible* ties, that is, among relational variables, irrespective of their observed values.

In building models for networks, a major step is to specify the form of local social neighborhoods, that is, the form of the regions within which contingent social processes are assumed to occur. The approach we describe below begins to construct a general framework within which hypotheses about neighborhood form can be developed and empirically examined. The foundation of the framework is the p^* class of models (Frank & Strauss, 1986; Pattison & Wasserman, 1999; Robins, Pattison & Wasserman, 1999; Wasserman & Pattison, 1996). The models are derived by assuming that possible ties that do not share a common locale are conditionally independent. The Hammersley-Clifford theorem (Besag, 1974) is then used to derive a general implied parametric form (see Frank & Strauss, 1986; Pattison & Wasserman, 1999; Wasserman & Pattison, 1996). The theorem establishes that non-zero parameters of a probability model for the network correspond to subsets of possible network ties that are assumed to be mutually conditionally dependent. By construction, each of these subsets corresponds to a collection of mutually contingent possible ties and corresponds to a local social neighborhood. In other words, the notion of social neighborhood is intrinsic to the p^* class of models.

The purpose of this paper is threefold. The first (Section 2) is to review some possible hypotheses about the form of local neighborhoods, including the Markovian assumption introduced by Frank and Strauss (1986). We argue that, while the theoretical rationale for this assumption is often strong, there are also reasons to consider both more restricted and more general assumptions about neighborhood form, and we note that certain of these lead to technical problems for the p^* class of models. Our second aim is therefore to propose two possible approaches to constructing plausible and more general classes of neighborhood assumptions, based on a more general notion of locale. In one approach (Section 3), we introduce the notion of *exogenous settings* that impose boundaries on possible neighborhoods. These exogenous setting structures can be used in a variety of ways to represent external constraints on possible network processes, for example to limit the assumed size of neighborhoods, or to represent spatiotemporal or external, organizational constraints. In the other approach (Section 4), we allow that neighborhoods can be created, in part, by the network processes themselves, and so can be seen both as a product of the interactive processes underlying the generation of network ties, and as the potential sites within which future processes are located. These developments depend on the introduction of the notion of partial conditional dependencies among network ties and we present a corresponding generalization of the Hammersley-Clifford theorem for the associated dependence structures. Our third aim is to present several applications of these approaches, and thereby demonstrate their potential value in the development of social network models (Section 5). We conclude with a brief discussion in Section 6 of parallel work that allows other forms of flexibility in neighborhood dependence structures, and with some speculations about future steps that might guide social network modeling.

2. The p^* class of models and local neighborhood hypotheses

A social network can be regarded as a designated set of nodes and their pairwise interconnections. In many applications, the nodes correspond to individuals and interconnections correspond to interpersonal ties (e.g., person *i* regularly communicates with person *j*), but, in general, a network may represent any collection of connections among a specifiable set of social units.

Let $N = \{1,2,...,n\}$ be a set of network nodes and let the two-way $n \times n$ (binary) array $\mathbf{x}$ denote an observed network on N: that is, $x_{ij} = 1$ if there is an observed tie from node *i* to node *j*, and $x_{ij} = 0$ otherwise. (See Appendix A for a full list of symbols.) Also let $\mathbf{X}$ denote a *random* graph or network on N, with each *possible tie* (i, j) regarded as a random variable X_{ij}. The possible tie (i, j) may be regarded as *directed* (and distinct from X_{ji}) or it may be *nondirected* (and identical to X_{ji}). We refer mainly to nondirected graphs in the discussion below.

As foreshadowed above, a potential problem with constructing models for $\Pr(\mathbf{X} = \mathbf{x})$ is that an assumption of independent variables is unlikely to be tenable; rather, ties are likely to be interdependent by virtue of shared locales. As a result, a modeling approach needs to give explicit recognition to possible interdependencies. Frank and Strauss (1986) recognized that some fundamental theorems for interdependent observations developed in spatial statistics could be applied to arbitrary dependence structures, including structures specifying assumed dependencies among network ties. Application of these results yields a general expression for $\Pr(\mathbf{X} = \mathbf{x})$ from a specification of which pairs of possible ties are conditionally independent, given the values of all other ties (Frank & Strauss, 1986).

More specifically, potential dependencies among possible network ties may be represented by a d*ependence graph* **D**. The node set M of the dependence graph indexes the set of possible ties[2], and the edge set $E = \{((i,j),(k,l))\text{: } X_{ij}$ and X_{kl} are conditionally dependent, given that $X_{mh} = x_{mh}$ for $(m,h) \in M \backslash \{(i,j),(k,l)\}\}$ specifies which pairs of possible ties are assumed to be conditionally dependent.[3] In the terms we have introduced above, possible ties are assumed to be conditionally dependent if they occupy a common locale, or neighborhood. Invoking the Hammersley-Clifford theorem (Besag, 1974) yields an expression for $\Pr(\mathbf{X} = \mathbf{x})$ in terms of parameters and sub-structures corresponding to cliques of **D**; that is

$$\Pr(\boldsymbol{X} = \boldsymbol{x}) = p^*(\boldsymbol{x}) = (1/c) \exp\{\Sigma_{A \subseteq M} \lambda_A z_A(\boldsymbol{x})\}$$

where:

(a) the summation is over all subsets A of M;

(b) $z_A(\boldsymbol{x}) = \Pi_{(i,j) \in A}\, x_{ij}$ is the *network statistic* corresponding to subset A of M;

(c) $c = \Sigma_{\mathbf{X}} \exp\{\Sigma_A \lambda_A z_A\}$ is a normalising quantity; and

[2] Often, $M = \{(i,j)\text{: } i, j \in N, i < j\}$ in the nondirected case, but, more generally, we allow that any subset of (i,j) combinations can define the set of possible ties.

[3] X_{ij} and X_{kl} are *conditionally independent*, given $X_{mh} = x_{mh}$ for $(m,h) \in M\backslash\{(i,j),(k,l)$ if

$\Pr(X_{ij} = x_{ij}, X_{kl} = x_{kl} \mid X_{mh} = x_{mh} \text{ for } (m,h) \in M\backslash\{(i,j),(k,l)) =$

$\Pr(X_{ij} = x_{ij} \mid X_{mh} = x_{mh} \text{ for } (m,h) \in M\backslash\{(i,j),(k,l)) \Pr(X_{kl} = x_{kl} \mid X_{mh} = x_{mh} \text{ for } (m,h) \in M\backslash\{(i,j),(k,l))$;

otherwise they are *conditionally dependent*.

(d) the parameter $\lambda_A = 0$ for all $\boldsymbol{x}$ unless A is a clique of $\mathbf{D}$ (where a *clique* of $\mathbf{D}$ is a nonempty subset A of M such that A comprises a single possible tie, or $((i,j),(k,l)) \in E$ for all (i,j), $(k,l) \in A$).

(See Frank and Strauss (1986) and Wasserman and Pattison (1996) for a detailed discussion of the models for nondirected and directed graphs, respectively.) This formulation can be generalized to both polytomous network data, where the entries in $\mathbf{X}$ can take a range of values signifying different strengths of tie (Robins et al, 1999), and to multivariate network data, where a number of edges of different types may be observed between pairs of nodes (Pattison & Wasserman, 1999).

Since it is assumed that possible ties are conditionally dependent only if they occupy a common social neighborhood, the subsets A for which the parameter λ_A is nonzero are all subsets of a local social neighborhood. (These subsets are themselves social neighborhoods.) It is in this sense that the form of social neighborhoods determines the form of the network model. The statistic $z_A(\boldsymbol{x})$ is a binary-valued measure corresponding to A that is computed from $\boldsymbol{x}$. It takes the value 1 if all of the possible ties in the subset A are present in $\boldsymbol{x}$, and 0 otherwise. The subset A corresponds to a subgraph configuration in $\boldsymbol{x}$, namely the subgraph obtained when all possible ties in A are present in $\boldsymbol{x}$. Thus, if the parameter λ_A is large and positive, the probability of observing the network $\boldsymbol{x}$ is enhanced if the configuration corresponding to A is present in $\boldsymbol{x}$ (net of other effects). More intuitively, if all ties in A are present (and so $z_A(\boldsymbol{x}) = 1$), then the model reflects – through the parameter λ_A – the effect of the network configuration on the probability of the network. Since the subsets of A are also neighborhoods, the model can represent the effects of higher-order, over and above lower-order, configurations. In principle, therefore, the model allows us to determine the extent to which lower-order building blocks contribute to the existence of higher-order configurations.

Markovian neighborhoods

A critical step in model construction is the specification of the form of local social neighborhoods, but what form should they take? Frank and Strauss (1986) introduced what we term here *Markovian neighborhoods*. A pair of possible ties X_{ij} and X_{kl} belong to a common neighborhood (and so are conditionally dependent) whenever they have a node in common, i.e., whenever $\{i,j\} \cap \{k,l\} \neq \varnothing$. This Markovian dependence structure specifies the independence, *conditional on the state of all other variables in the network*[4], of any two variables X_{ij} and X_{kl} that correspond to possible ties with no node in common.

The assumption of Markovian neighborhoods reflects a natural intuition about processes that generate social relationships: social ties are not independent of each other, but their dependence is expressed through any persons directly involved in the ties in question. The assumption is also a natural graph-theoretical generalization of the dyad-independence assumption, since it asserts that dependencies arise not just in potential tie cycles of length two (that is, in dyadic structures), but also in any potential semi-path of length two[5].

For nondirected graphs, the configurations corresponding to Markovian neighborhoods take one of a relatively small number of forms: edges; triads; and star-like structures (see

[4] In contrast to dyad-independent models, the conditionality is necessary, for if X_{ij} is not independent of X_{jk}, and in turn X_{jk} is not independent of X_{kl}, then X_{ij} cannot be said to be independent of X_{kl}, even if $\{i,j\} \cap \{k,l\} = \varnothing$. Conditional dependence here is, in fact, equivalent to the claim that X_{ij} and X_{kl} are independent, given the states of variables variables X_{ik}, X_{ki}, X_{jk}, X_{kj}, X_{il}, X_{li}, X_{jl} and X_{lj}, that is, the variables in the intersection of the maximal neighborhoods of the possible ties (i,j) and (k,l). More generally, once the state of variables in the neighborhood of (i,j) is known, then the state of other variables provides no information about X_{ij}. In fact, it is possible to construct some artificial examples where conditionality on all other variables and conditionality based on variables corresponding to the intersection of neighborhoods are not equivalent. These examples typically arise because of a logical relation among the variables, which will not be the case in the types of networks that we consider. See Lauritzen (1996) for further discussion.

[5] A *semi-path of length k* in a directed graph is a set $\{(i_1,i_2), (i_2,i_3),\ldots,(i_k,i_{k+1})\}$ of node pairs such that either (i_h,i_{h+1}) or (i_{h+1},i_h) is an edge in the graph, for each $h = 1,2,\ldots, k$. A *cycle* of length k is a set $\{(i_1,i_2), (i_2,i_3), \ldots, (i_k,i_1)\}$ of edges linking k distinct nodes $i_1, i_2,\ldots, i_k$
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Figure 1). If a homogeneity assumption is also made (that parameters corresponding to isomorphic neighborhoods are equal)[6], then the probability model for $\boldsymbol{X}$ has a single parameter corresponding to each of the distinct configurations in Figure 1. Each configuration corresponds to an isomorphism class $[A]$ of neighborhoods A, and the statistic $z_{[A]}(\boldsymbol{x})$ corresponding to class $[A]$ is then a count of the number of observed configurations of that form. The model for the global network structure $\boldsymbol{X}$ expresses the probability of a network in terms of propensities for these triadic and star-like configurations to occur.

Insert Figure 1 about here

Model parameters are often currently estimated using a maximum pseudo-likelihood procedure (Strauss & Ikeda, 1990), but other Markov Chain Monte Carlo estimation approaches are also under investigation (e.g., Corander, Dahmström & Dahmström, 1998; Crouch & Wasserman, 1998, Snijders, 2001). Computational details of the maximum pseudo-likelihood approach for Markovian and other models considered here are presented in Appendix B.[7]

Compared to models with only edge and dyadic neighborhoods, empirical support for models with Markovian neighborhoods has been strong in a number of applications involving a broad range of network types (e.g., Lazega & Pattison, 1999; Pattison & Wasserman, 1999;

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[6] In general, two configurations A and A' are *isomorphic* if there is a 1-1 mapping φ on N such that $(i,j) \in A$ iff $(\varphi(i),\varphi(j)) \in A'$). If we assume that $\lambda_A = \lambda_{A'}$ whenever A and A' are isomorphic, the model then takes the form $\Pr(\boldsymbol{X}=\boldsymbol{x}) = (1/c)\exp\{\Sigma_{[A]}\lambda_{[A]}n_{[A]}(\boldsymbol{x})\}$, where $[A]$ is the class of cliques in **D** isomorphic to A, and $n_{[A]}(\boldsymbol{x}) = \Sigma_{A\in[A]}z_A(\boldsymbol{x})$ is the corresponding *sufficient statistic*.

[7] The development of feasible methods for maximum likelihood estimation is an area of active research (for example, Corander et al., 1998; Crouch & Wasserman, 1998; Snijders, 2001). Preliminary results suggest that it is useful to pursue the development of more plausible models for networks in parallel with this work, since the performance of estimators is critically dependent on model properties.

Robins, 1998; Robins et al., 1999; Wasserman & Pattison, 1996). This support is not surprising in the light of strong theoretical claims about triadic dependencies such as those reviewed earlier; indeed, empirical findings have already mirrored aspects of balance theory (e.g., Pattison & Wasserman, 1999), Granovetter's strength-of-weak ties hypothesis (e.g., Robins et al, 1999), and local clustering effects (Lazega & Pattison, 1999). Yet whether Markovian neighborhoods provide the most precise specification of local neighborhoods for network structures is uncertain, and below we consider the theoretical task of evaluating the necessity and sufficiency of a Markovian formulation.

Beyond Markovian neighborhoods

It is easy to conceive of hypothetical situations in which the generic Markovian neighborhoods that we have just described are at once too broadly and too narrowly specified. On the one hand, it is possible that the ties X_{ij} and X_{kl} arise within a common social locale even though i, j, k and l are all distinct (for instance, they may all meet regularly in a group setting, or be connected by common interests), and they may, accordingly, be conditionally dependent. On the other hand, X_{ij} and X_{ik} may never occupy the same settings: even though individual i is common to the two possible ties, the possible tie (i, j) may occupy a distinct set of social locales to those containing the possible tie (i, k), and, as a result, it is plausible that the two possible ties are, in fact, conditionally independent of one another. Indeed, in large networks, individuals may not be aware of all of the possible ties of those to whom they are themselves tied, nor even of all other nodes in the network; in addition, contextual factors may be such that certain potential Markov dependencies are never realized. In other words, these hypothetical examples raise the possibility that the generic Markovian neighborhoods introduced above do not map directly onto an as-yet-unspecified structure of social locales that underpin conditional dependencies among variables.

Below, we consider two possible ways of constructing more general, and arguably more plausible, specifications of neighborhood form. In the first, a setting structure is directly hypothesized (or even observed) and considered as a set of exogenous constraints on the model. These assumed exogenous constraints take the form of substantive limitations on possible tie interactions and are used to hypothesize directly that certain model parameters are zero. In the second, neighborhoods are proposed to be generated, in part, by the unfolding interactive processes themselves. In this second case, the neighborhood structure for the model of a system of tie variables depends on the *realization* of the model; that is, whether two possible ties are conditionally dependent may depend on whether a particular subset of possible ties is actually observed to be present.

Before describing these two approaches to modifying a generic Markovian neighborhood specification, we note that a simple generic extension of the Markovian assumption is problematic for the p^* class of models. In particular, if we allow that X_{ij} and X_{lm} are conditionally dependent for all $((i,j),(l,m)) \in E$, then the dependence graph **D** is complete. In this case, every subset of nodes in M corresponds to a neighborhood, and even with a general homogeneity assumption, the resulting p^* model cannot be estimated. Of course, in order to render the model estimable, one can choose to set particular parameters to zero, and the first approach that we describe below provides a way of proposing a principled set of choices. Our second approach introduces a modified form of conditional dependence assumption and so arguably retains a more explicit link between conditional dependence assumptions and model form. In the applications presented in section 5, both approaches are used.

3. **Setting structures**

The first approach is to hypothesize a setting structure directly. That is, we assume that possible ties are conditionally dependent only when they share a potential "site of social action" (White, 1995a) or *social setting* (also, Feld, 1981). Each setting is assumed to correspond to some subset of possible network ties.[8] The notion of setting is intended, for the moment, to be a very general and skeletal one. It may correspond to some spatiotemporal context, such as a group of people gathered together at the same time and place, or it may refer to a collection of possible ties that are connected in some more abstract sociocultural space (for instance, pairs of persons linked by their political commitments). Settings may even reflect some external "design" constraints, such as organizational structure, task requirements in organizational settings, or hardware capabilities in communication networks. We have deliberately chosen an abstract formalization of the concept of setting, given the current limited state of theoretical and empirical articulation (but see Mische & Pattison, 2000). And, for the moment, we simply propose that settings impose limits on local neighborhoods. Like neighborhoods, they may, in general, overlap, and possible ties may occupy many neighborhoods within many settings simultaneously. But mutual conditional dependencies among a set of ties (that is, neighborhoods) are assumed to be restricted so that they occur only within common settings. That is, two possible ties are assumed to be conditionally independent if they do not occupy a common setting, and they may or may not be conditionally independent if they do. Thus, a setting is a weaker notion than a

[8] In fact, although we do not pursue the possibility here, it is likely that these subsets of possible ties are associated with socio-cultural locales that include jointly produced interpretive frames through which actions of the participants can be understood. White (1995a,b) coined the term *network-domain* to refer to this inter-coordination of sets of (possible) network ties and cultural domains. He proposed that these network-domains as well as *switchings* between them are the primitive entities through which the regularities and discontinuities in socio-cultural processes might come to be understood.

neighborhood, since the membership of two possible ties in a common setting is only a necessary but not sufficient condition for conditional dependence.

Formally, we define a *setting* $s \subseteq M$ as a subset of possible ties on a set N of nodes and a *setting structure* S simply as a collection of settings on N. We assume that if s is a setting in S, then so is any subset of s. Thus, we can think of a setting structure is a *closed hypergraph*[9] on the set M that indexes all possible ties on the node set N.

Suppose that **X** is a random network with generic dependence structure **D** whose edge set is E (for example, **D** may assume Markovian neighborhoods). Also, let H denote the set of all cliques of **D**; H is also a closed hypergraph on E (Robins, 1998). Then a random graph model *confined* by the setting structure S has *the setting-restricted* clique set $H_S = H \cap S$; H_S is also a closed hypergraph on E.[10] Thus, in the restricted dependence structure, a collection of possible ties is mutually conditionally dependent only if it lies in some common setting. A setting structure hypothesis thus provides one approach to setting parameters in a model to zero: by restricting the clique set of the dependence graph to H_S, we impose the assumption $\lambda_A = 0$ for any $A \notin H_S$.

For a given dependence structure, setting structure hypotheses and homogeneity assumptions are usually both invoked. These two sets of assumptions are not completely independent, however, as a setting structure hypothesis may set the parameters corresponding to some members of an isomorphism class [A] to zero, but not others. We assume below that

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[9] A *hypergraph H* consists of a set V of elements and a collection E of subsets of V, termed edges. It is assumed that each element of V belongs to at least one edge, and that no edge is empty (e.g., Berge, 1989). The hypergraph is *closed* if every subset of an edge in E is also an edge.

[10] The closure of H and H_S ensures that the corresponding probability models are hierarchical.

setting structure hypotheses have primacy, that is, that the homogeneity assumption applies to all cliques $A \in H_S$.

Several different forms for possible setting structures can be distinguished. If S comprises a single setting corresponding to the set M of all possible ties, then it is termed *universal*. In this case, the setting-restricted clique set H_S is simply the clique set H associated with the generic dependence structure **D**.

Setting structures may also be defined for disjoint groups. Suppose that $N = \cup_g N_g$ is a disjoint union of the node set N, and let S_g be the universal setting structure defined on N_g. Then $S = \{S_g\}$ defines a disjoint subgroup structure and the model for $\Pr(\boldsymbol{X} = \boldsymbol{x})$ can be decomposed into the form $\Pi_g \Pr(\boldsymbol{X}_g = \boldsymbol{x}_g)$, where $\boldsymbol{X}_g$ denotes the random network on the node set N_g, and $\boldsymbol{x}_g$ is its corresponding realization.[11] Such a form was described by Anderson, Wasserman and Crouch (1999). Note that homogeneity of parameters corresponding to isomorphic neighborhoods may be assumed either within-groups, or both within- and between groups. Indeed, comparisons between models making these different homogeneity assumptions may yield useful insights into the heterogeneity of parameters across groups, as Anderson et al (1999) demonstrated.

More generally, it may be appropriate to regard settings as group-like in structure but potentially overlapping (e.g., Freeman & White, 1993; Mische, 1998). In particular, if we conceptualize a setting in terms of the potential links among a subset N_s of individuals who are, say, co-present in a particular location, co-members of a particular group, or who share

[11] In this case it is also possible to think of the restriction arising at the level of the dependence graph, not just at the level of its cliques. That is, if we define a *setting-restricted dependence graph* $\mathbf{D}_S$ with edge set $E \cap \{\cup_g(M_g \times M_g)\}$, where M_g is the set of possible ties on N_g, then the clique set of $\mathbf{D}_S$ is simply H_S. The restriction acts to ensure that only pairs of possible ties from the same group give rise to an edge in $\mathbf{D}_S$.

certain aspirations, then settings take the form $s_s = \{(i,j): i, j \in N_s \text{ and } i \neq j\}$ and a potentially *overlapping subgroup setting structure* results.

Versions of overlapping subgroup setting structures can be used to explore the interaction between neigbourhoods specified in network terms (as in the Markovian case) and neighborhoods defined in terms of regions in physical or social space. For example, as White (1992) has argued, physical space is likely to be important to the notion of locale, and it is important to develop an understanding of the relationship between physical space and local neighborhoods. If we assume that individuals have some spatial distribution, we can use spatially-defined setting structures of different forms and extents to explore the properties of the very different neighborhood structures that network- and spatially-restricted neighborhoods imply.

An overlapping subgroup structure also arises if we assume that settings are restricted to a maximum of k individuals, so that each setting comprises ties among a subgroup of individuals of size no greater than k. Such setting structures are implicit when we restrict neighborhoods according to the number of nodes that they involve. For instance, with a generic Markov assumption, and $k \geq 3$, such a setting structure leads to a model in which star configurations involving more than k nodes have zero parameters, a model that has proved useful in many empirical applications for $k = 3$ (see also Robins, 1998).

To illustrate how overlapping subgroup setting structures may be invoked, we use organizationally-defined overlapping subgroups to set certain model parameters to zero in one of the applications presented in Section 5. As indicated above, the parameters are set to zero prior to the imposition of a homogeneity constraint on isomorphic within-setting forms.

More generally, a setting structure comprises a collection of subsets of possible ties, subject to the requirement that each subset of a setting is also a setting. For example, a general setting structure form that has proved technically useful, especially in multivariate applications, is one in which the number of ties in a setting is restricted to some maximum (e.g., see Lazega and Pattison, 1999).

4. Partial conditional dependence assumptions

It was observed above that the general assumption that X_{ij} and X_{kl} are conditionally dependent for distinct i, j, k, l leads to a complete dependence graph and associated problems. In fact, though, such a general assumption is likely to be unreasonable in any but the smallest face-to-face group contexts. Rather, if there is a conditional dependence between X_{ij} and X_{kl} for distinct i, j, k, l, it is likely to result from the fact that these possible ties occupy a common setting. In the section above, we introduced hypothesized setting structures as a means of using information about settings to restrict generic dependence assumptions. In this section, we consider the possibility that the interactive processes giving rise to network ties are themselves a source of social settings, so that new settings are created as network ties are generated: for instance, the possible ties X_{ij} and X_{kl} might become conditionally dependent if there is an observed tie between j and k or between l and i.

In other words, we might assume that longer-range dependencies (that is, those involving X_{ij} and X_{kl} for distinct i, j, k, l) are restricted by such conditions as:

$$((i,j),(k,l)) \in E \text{ for distinct } i, j, k, l \text{ only if } x_{ik} = 1 \text{ or } x_{il} = 1 \text{ or } x_{jk} = 1 \text{ or } x_{jl} = 1.$$

We noted above that the Markov assumption introduces conditional dependencies among possible ties forming semi-paths of length two, and that this can be seen as a natural graph-theoretical extension of the assumption of dependence among possible ties forming cycles of length two.[12] The condition just posited extends the assumption of conditional dependence

[12] In fact, an intermediate model between the dyad-independent model and the Markov model is the *(2)-*

among possible ties forming a path of length 3 (and the middle tie must be present for the assumed conditional dependence among the first and last possible ties in the path).[13]

The condition introduces the notion of *partial* conditional independence with respect to some subset of ties: X_{ij} and X_{kl} are conditionally independent for certain observed values of other variables (e.g.if $x_{ik} = 0$ and $x_{il} = 0$ and $x_{jk} = 0$ and $x_{jl} = 0$) but conditionally dependent for certain other observed value combinations (e.g., if $x_{ik} = 1$ or $x_{il} = 1$ or $x_{jk} = 1$ or $x_{jl} = 1$).

To capture the idea in general, we define a *partial dependence structure* $\mathbf{D}_B$ for a subset $B \subset M$ of possible ties. The node set of $\mathbf{D}_B$ is the set $M \backslash B$ of possible ties that are in M but are not in B and the edge set of $\mathbf{D}_B$ is given by $\{((i,j),(k,l))$: X_{ij} and X_{kl} are conditionally dependent, given that $X_{mh} = x_{mh}$ for $(m,h) \in M \backslash B$ and $X_{mh} = 0$ for $(m,h) \in B\}$. In other words, two possible ties are linked by an edge in $\mathbf{D}_B$ if they are assumed to be conditionally dependent even when all of the possible ties in the set B have observed values of 0. More importantly, two ties are *not* linked by an edge in $\mathbf{D}_B$ if they are assumed to be conditionally independent when all of the possible ties in the set B have observed values of 0. Note that if $((i,j),(k,l)) \in \mathbf{D}_B$ then $((i,j),(k,l)) \in \mathbf{D}$; thus, $\mathbf{D}_B$ is a subgraph of $\mathbf{D}$ for all $B \subset M$. It is possible though that $((i,j),(k,l))$ may be an edge in $\mathbf{D}$ but not in $\mathbf{D}_B$, for some B, signifying that X_{ij} and X_{kl} are conditionally independent when $x_{mh} = 0$ for all $(m,h) \in$ B. We refer to two conditionally dependent possible ties as *partially conditionally independent* with respect to a

path model described by Pattison and Wasserman (1999), in which X_{ij} and X_{kl} are conditionally dependent whenever $j = k$, that is, when X_{ij} and X_{kl} form a possible 2-path.

[13] It is interesting to note that the construction we propose here is similar to that suggested by Baddeley and Möller (1989) who were concerned to formulate spatial interaction models for randomly positioned objects in a way that would satisfy certain geometrical or topological conditions but also retain good probabilistic and statistical properties.

set of possible ties B (a set that includes neither tie) if the pair of ties represents an edge in **D** but not in $\mathbf{D}_B$.[14] We show in Appendix C that:

Proposition:

If $A \subseteq M \backslash B$ and A is not a clique in $\mathbf{D}_B$ for some subset B of M, then $\lambda_A = 0$ in $\Pr(\boldsymbol{X} = \boldsymbol{x})$.

In other words, if there exists *any* subset B of possible ties for which $A \cap B = \varnothing$ and the set A of possible ties is not a clique in $\mathbf{D}_B$, then the parameter λ_A corresponding to A must be zero. It therefore follows that

Corollary:

The parameter λ_A is non-zero in the model $p^*(\boldsymbol{x})$ if and only if A is a clique in **D** and in all $\mathbf{D}_B$ for which $A \cap B = \varnothing$.

For example, suppose, as suggested above, that in addition to the assumption that X_{ij} and X_{kl} are conditionally dependent if $(i,j) \cap (k,l) \neq \varnothing$, we assume that X_{ij} and X_{kl} are conditionally dependent for distinct i, j, k and l if $x_{ik} = 1$ or if $x_{il} = 1$ or if $x_{jk} = 1$ or if $x_{jl} = 1$ but are conditionally independent otherwise. Then $((i,j),(k,l))$ is an edge in **D** but $((i,j),(k,l))$ is not an edge in $\mathbf{D}_B$, for $B = \{(i,k),(i,l),(j,k),(j,l)\}$. Thus, for $A = \{(i,j),(k,l)\}$, $\lambda_A = 0$ even though for $A = \{(i,j),(j,k),(k,l)\}$, λ_A may be non-zero. It follows from this conditional dependence structure that non-Markovian parameters correspond to certain connected configurations

[14] Partial conditional independence is a generalization of the more familiar notion of conditional independence. Conditional independence is the expression of the statistical independence of two variables given the state of a third variable (Dawid, 1979, 1980). In the case of partial conditional independence, we assert that two variables are statistically independent if and only if a third variable is in a *particular* state.

comprising 4 or more nodes that satisfy the condition that every pair of edges lies on a path of length 3. We refer to this model as the *3-path* random graph model. For non-directed graphs, there are non-zero model parameters corresponding to configurations in which every pair of edges lie on a path of length 3; configurations on 4 nodes satisfying the condition are shown in Figure 2. Note that these configurations contain longer paths and cycles than the Markov configurations of Figure 1.

Insert Figure 2 about here

A positive model parameter corresponding to one of these higher-order configurations would reflect a tendency for networks to be more probable if they possess many such higher-order configurations. For example, a model with a positive parameter for a 3-path (the first configuration in Figure 2) would indicate that, given a particular tendency towards edges and 2-paths, networks with many 3-paths are more likely. In example 2 below, we also suggest how models with positive parameters for 4-cycles (the third configuration in Figure 2) might be interpreted.

For directed graphs, both *3-semi-path* and *3-path* random directed graph model can be formulated, with two possible ties conditionally dependent if they lie on a 3-semi-path, or 3-path, respectively. In the applications presented below, we use the 3-path model for nondirected graphs; for directed graphs, though, it is an empirical question as to which might provide the most useful model in a given modelling context.

The steps just described allow us to embed models with Markovian neighborhoods in classes of models with more complex dependence structures. This embedding has two important theoretical consequences. First, it permits a more detailed assessment of the class of Markovian models. Only by comparing Markovian models with models that make plausible but more complex neighborhood assumptions can we explore the sufficiency of the Markovian neighborhood assumption. Second, it permits us to examine some more complex dependencies that are suggested by several theoretical claims, including: arguments for the presence of *generalized exchange* and hence for *cyclic* patterns of network ties (e.g., Bearman, 1997); arguments about the nature of strategic activity in networks, including brokering and mediating behaviours (e.g., Mische, 1998, Mische & Robins, 2000); and arguments for indirect social influence (e.g., Robins, Pattison & Elliott, in press).

In theoretical terms, it is worth emphasizing that extra-Markovian dependence assumptions allow us to explore the possibility of "action at a distance", whereby individual ties may be shaped by events or opinions that are not contiguous to the individuals concerned. For instance, *generalized exchange* (Bearman, 1997; Breiger & Ennis, 1997; Yamagishi & Cook, 1993), requires extra-Markovian neighborhood assumptions. Although exchange *within* dyads is often governed by a norm of reciprocity, *generalized exchange* is governed by reciprocity with a difference: those who receive also give but they give to someone else. The result is a system of cycles where "values have to flow through all parties in a cycle before a giver can become a taker, that is, receive a gift in return" (Bearman, 1997, p.1389). A Markov assumption is adequate to model cycles of order three (e.g., see Pattison & Wasserman, 1999) but Bearman's analysis revealed cycles of order 8 in a generalized system of marital exchange within aboriginal kinship systems. To investigate the relevance of cyclic structures within a network, we need to assume that a possible tie is conditionally dependent on other possible ties with which it might form cycle-constituting paths. Thus, partial conditional dependence assumptions such as those described above are required. An example of this approach for cycles of order 4 is presented in the applications below.

A final observation on partial conditional dependence structures is that the associated models may not be hierarchical. We observed earlier that the cliques of a dependence graph define a closed hypergraph, since, for a dependence structure, every subset of a clique is necessarily also a clique. Similarly, setting-confined clique sets are also closed hypergraphs by virtue of the construction of setting structures. In the case of models associated with partial conditional dependence structures, however, it is possible for a model to have a non-zero parameter corresponding to λ_A, even though $\lambda_{A'}$ is fixed to be zero for some $A' \subset A$. In this case, the collection of cliques with non-zero parameters define a hypergraph that is not closed. The interpretation of the parameter λ_A is made relative to constituent substructures A' for those A' that correspond to connected configurations and that are included in the model (rather than for all such A'). This point is illustrated in the interpretation of models for the last two applications below.

5. **Applications**

We present two applications that draw on the approaches to neighborhood formulation that we have described.

Friendship setting structures from organizational constraints

In the first example, we use a familiar network data set to illustrate the application of an overlapping subgroup setting structure. The data come from Roethlisberger and Dickson's (1939) classic study, *Management and the Worker*, and we use some of the information about work arrangements supplied by the authors and discussed by Homans (1951) to model the network of friendships in the Bank Wiring Room.

As both Roethlisberger and Dickson and Homans report, the 14 workers in the Bank Wiring Room occupied particular locations in the room and their work arrangements dictated certain functional interdependencies. Here we use the two types of organizational constraints illustrated in Homan's (1951, p. 56) Figure 1 to generate setting structure hypotheses. The first is the arrangement of workers into soldering units. Each unit comprised 4 workers, with 3 engaged in wiring and one in soldering. The units were (W1, W2, W3, S1), (W4, W5, W6, S2) and (W7, W8, W9, S3). A slightly different structure was associated with work inspections, with inspection units comprising (W1, W2, W3, S1, W4, W5, W6, I1) and (W5, W6, S2, W7, W8, W9, S3, I2).

Here we assume that the setting structure for the Bank Wiring Room comprises the collection of these subgroups as well as all pairs of workers in the room. The rationale for this hypothesis is twofold. First, friendship ties are possible among all members of the room, hence we include all possible ties in the setting structure. Second, more complex dependencies involving two or more possible friendship ties are most likely to have their source in the likely more intensive work-related interactions among members of a work unit; accordingly, we include functional work units in the setting structure. Of course, there may be social settings outside of work that contribute to possible tie dependencies and that are not captured by this setting structure; such dependencies are not modelled by this hypothesized structure, and are likely to be associated with lack of fit.

In Table 1, we report the fit of 5 models to the Friendship network in the Bank Wiring Room (the data are presented in Homans, 1951, p. 69, Figure 6). The first is the homogeneous Bernoulli model and it is presented for comparative purposes. (The model assumes a generic dependence structure in which all possible friendship ties are independent, so that the cliques in *H* correspond to edge configurations and each has the same parameter.) For each of the fitted models, Table 1 shows two heuristic indices of model fit: -2 times the log of the maximized pseudolikeliohood function (-2LPL: see Appendix B); and the mean absolute residual (MAR) for each possible tie).[15] We rely on heuristic measures of fit since the distribution of –2LPL is not known (as for other applications of maximum pseudolikelihood estimation; see Besag, 1977, and Strauss & Ikeda, 1990). The pseudolikelihood parameter estimates are also shown in Table 1; computational details are given in Appendix B. The second and third rows of the table show two Markov models for a universal setting structure; the fourth and fifth rows show the same models confined by the setting structure described above. (Note that additional restrictions are imposed in both cases, since only a subset of possible Markov configurations are used).

Insert Table 1 about here

It is evident from the table that the inclusion of a 2-star parameter improves model fit in the presence of the restricted setting structure hypothesis, but that the improvement is only marginal if the setting structure is assumed to be universal. Comparison of parameter estimates shows that the edge parameter is estimated to be substantially more negative and the star parameter substantially more positive for the restricted setting structure case. In the restricted set of neighborhoods associated with the setting structure hypothesis, therefore, there appears to be a tendency for friendship ties to be connected through particular individuals, and for isolated friendship ties to be rare.

[15] The mean absolute residual is computed as the average value of $|x_{ij} - z_{ij}|$, where z_{ij} is the estimated value of $Pr(X_{ij} = 1 \mid X_{mh} = x_{mh} \text{ for } (m,h) \in M \backslash \{(i,j)\})$, computed from the conditional logit form of the p^* model, namely, logit $Pr(X_{ij} = 1 \mid X_{mh} = x_{mh} \text{ for } (m,h) \in M \backslash \{(i,j)\}) = \Sigma_B \lambda_B \Pi_{X_{klh} \in B \backslash \{X_{ij}\}} x_{klh})$, where B is the set of substructures including the possible tie X_{ij}.

For models possessing both 2-stars and triangles, there is less difference in the fit of the models with universal and restricted setting structures, and also a less clear difference in parameter estimates. The largest difference is in the 2-star parameter, which is close to zero when neighborhoods are setting-restricted and small and negative without the setting restriction. It is interesting that the effects of the setting-restriction are so substantially moderated once the triangle parameter is introduced. It is possible that, in this case, the triangle parameter largely has its effect within settings, so that the setting structure hypothesis and the triangle (clustering) effect are somewhat redundant. In other words, tendencies towards clustering in the Bank Wiring Room may arise principally from the organizational division of labor. This simple example suggests that formal organizational structure may play an important role in the evolution of informal networks, with balance and clustering effects to some degree constrained by formal boundaries.

Work organization in a New England law firm

The second application is to data from a study conducted by Emmanuel Lazega in a US law firm (see Lazega, 1993; Lazega & van Duijn, 1997; Lazega & Pattison, 1999). All 71 lawyers in the firm were interviewed and provided information about a number of network ties among firm members. One of the questions sought information about Work ties in the firm.[16] Here we consider the matrix of reciprocated work ties, in which $x_{ij} = 1$ if both i and j claim to work with each other, and $x_{ij} = 0$, otherwise. Since the ties are non-directed, Markovian neighborhoods correspond to edges, k-stars (for $2 \leq k \leq 70$) and triangles; see Figure 1.

[16]The question was: “Here is the list of all the members of your Firm: Think back over the past year and check the names of the lawyers with whom you have worked. By ‘worked with’ I mean that you have spent time together on at least one case, that they have read or used your work product, or that you have read or used their work product; this includes professional work done within the firm like Bar Association work, administration, etc.”

The purpose of our analyses is to examine the structure of these reciprocated Work ties, and to ask (a) whether they possess any regular, discernible form of local structuring, and (b) whether Markovian neighborhoods are sufficient to describe any such local regularities. From a substantive point of view, any regularities in the social organization of Work ties are of considerable interest. On the one hand, Work ties are not centrally organized in a professional group such as a law firm, and are clearly somewhat contingent on lawyers' availability and expertise. On the other hand, the firm and its members are critically dependent on a timely flow of work for their financial well-being. Thus, the question of how work relationships come to be forged is a critical one, and regularities in the organization of work ties may suggest the emergence of social forms that serve to ameliorate this problem.

In the first two rows of Table 2 we report the fit of homogeneous Bernoulli and Markov models. In fitting the homogeneous Markov model, we assume that parameters corresponding to higher-order stars (i.e., $k > 3$) are zero, and so we invoke the assumption that settings comprise no more than four lawyers.[17] As comparison of the heuristic indices of fit indicates, the Markov model provides a substantial improvement over the Bernoulli model, and the estimated values of its parameters suggest that reciprocated Work ties exhibit quite substantial local clustering (parameter estimates appear in the top panel of Table 3). It is worth noting also that both star parameters have estimated values close to zero, indicating that there is no particular tendency for or against the generation of ties in ways that create 2-stars or 3-stars. The lack of a tendency for or against 2- or 3-stars suggests that there is no particular tendency for or against work partners with high degree. In this work network, in other words, there is little evidence for high levels of centralization. Further, the low values of the star parameters suggest that the local clustering tendency is not accompanied by a

[17] See Robins et al (1999) for a discussion of the role of higher-order stars in Markov models.

tendency against unclosed 2-paths, leading to an overall structure with the character of a triangular tessellation.

Insert Tables 2 and 3 about here

In the third row of Table 2, we present the fit of the model in which we assume that the variables X_{ij} and X_{kl} are conditionally dependent if either (a) $\{i,j\} \cap \{k,l\} \neq \emptyset$ (as in the Markovian case) or (b) $\{i,j\} \cap \{k,l\} = \emptyset$ and x_{ik} or x_{il} or x_{jk} or x_{jl} is equal to 1; otherwise, we assume that X_{ij} and X_{kl} are conditionally independent. The resulting partial dependence assumptions lead to neighborhoods that correspond to edges, stars, triads and higher-order structures such as those shown in Figure 2. If we impose a setting structure of level 4, then the parameters of the model correspond to configurations with no more than 4 edges. Several of the parameters corresponding to these configurations are close to zero; their elimination leads to the model shown in the fourth row of Table 2; parameter estimates are given in Table 3. As the heuristic fit indices suggest, this model is an improvement over the Markov model, and the parameter corresponding to a 4-cycle of reciprocated Work ties is positive, suggesting that the probability of a Work network is enhanced by the presence of both 3-cycles and 4-cycles. Consistent with the interpretation made by Lazega and Pattison (1999) in their analysis of directed Work ties, it appears that these results provide some evidence for the presence of a form of generalized exchange in the arrangement of Work ties. The cyclic structures do not encompass the entire group, as in the case of the Groote Eylandters analysed by Bearman (1997); rather the 3-cycle and 4-cycle structures occur locally, and they may overlap with one another. It is also worth noting that the three-path parameter is negative. This implies that long unclosed paths are relatively rare, and that those 3-paths that are found

are more likely to arise in local cyclic structures of length 4. It should be noted that the evidence is weak: in order to make a stronger claim, it would be useful to compare the model in the third and fourth rows of the table with one assessing the role of more densely connected subsets of 4 nodes, as well as with one possessing longer cycles.
Nevertheless, the ability of this model to represent effects involving longer cycles provides a new empirical method for investigating generalized exchange.

6. **Prospects**

In this paper, we have argued that social networks may be modeled as the outcome of processes that occur in overlapping local regions of the network, termed local social neighborhoods. Each neighborhood is conceived as a possible site of interaction and corresponds to a subset of possible ties. We have also argued that while there is growing evidence for the value of Markovian neighborhoods for network models, there are reasons to doubt the universal applicability of the Markovian neighborhood assumption. Accordingly, we presented two theoretically plausible ways in which Markovian and other neighborhood assumptions can be modified. The first is to introduce the notion of a setting structure – a directly hypothesized (or observed) set of exogenous constraints on possible neighborhood forms. The second is to propose higher-order neighborhoods that are generated, in part, by the outcome of interactive network processes themselves.

The two constructions that we introduced were deliberately presented at a very general level, since we believe we are not yet in a position to make strong claims about higher order forms that are likely to be useful in a wide range of network modeling settings. Nonetheless, even though the applications of these constructions that were presented were illustrative only, they do point to the potential theoretical value of more carefully crafted hypotheses about network neighborhoods. In this final section, we briefly review the applications that we have presented and then suggest some potentially important additional applications. We also sketch some next steps in the exploration of neighborhood forms.

Non-Markovian neighborhoods

The applications provide some initial evidence for the empirical value of models with non-Markovian neighborhoods. Higher-order structures in the organization of work in the law firm are both theoretically meaningful and lead to a substantially enhanced capacity to account for network data. In relation to substantive setting structure hypotheses, we have only explored one limited application to the small network of friendship ties in the Bank Wiring Room: clearly, more extensive investigations, particularly in large networks where setting effects are likely to be more pronounced, are required. Nonetheless, the interesting pattern of results obtained from the Bank Wiring Room application suggests a possible interaction between setting structures and neighborhood forms. It may be the case that setting structure hypotheses will prove most useful where neighborhood assumptions are minimal and where networks are large. It may also be the case that the major impact of setting structure hypotheses of the type investigated for the Bank Wiring Room is for lower-order neighborhood forms, such as stars. Higher-order forms (such as triangles) are most likely to occur within settings anyway, and so to be somewhat redundant with them, whereas lower-order forms may be implicated in structures that bridge settings.

Within a general conceptual framework of the social locales underpinning social networks, we have presented various dependence assumptions as a hierarchy of increasingly complex dependence structures. The development of partial conditional dependence structures allows the fitting of models based on local configurations with increasing capacity to “reach” across the network through large cycles or long semi-paths. In a graph-theoretic sense, we can represent dependencies among possible ties within a hierarchy of structures with increasingly longer “reach”. The hierarchy of models is presented in Table 7.

Insert Table 7 about here

Further applications

Many additional applications of the constructions that we have introduced have been suggested. For example, Mische and Robins (2000) use partial conditional independence assumptions in modeling tripartite networks linking youth leaders, social and political organizations, and political events during the 1992 Brazilian impeachment movement. The resulting higher-order neighborhoods allow them to examine hypotheses about the roles of youth acting as mediators, brokers or co-ordinators between organizations.

In addition, these constructions are especially useful in a number of models that distinguish classes of variables and permit certain directed dependencies between variables in different classes. For example, Robins, Pattison and Elliott (in press) constructed social influence models that examine contingencies between attributes of nodes and social organization. In these models, network ties and individual attributes form two distinct variable classes, and dependencies are assumed to be directed from the tie variables to the attribute variables. Mutual within-class dependencies are also assumed. These developments are noteworthy for two reasons. First, the neighborhoods associated with such models comprise variables that refer to potentially interdependent phenomena of different types; in principle, for instance, they might refer to such entities as possible ties, actor attributes, group memberships and discursive forms. As a result, these models lead to a generalization of the notion of neighborhood from a collection of possible ties to more complex and, arguably more realistic, sociocultural forms – social locales, in the sense used earlier (Mische & Pattison, 2000). A second important feature of these models is that they can instantiate the assumption of influence-at-a-distance (e.g., that the attributes of individuals *i* and/or *j* can influence the attribute of a third individual *k*). Such models require a partial conditional independence assumption. For example, it may be hypothesized that such effects are likely only when certain subsets of the network ties among *i*, *j* and *k* are present. Similar approaches can be applied to social selection models, where the attributes of individuals are assumed to shape the formation of network ties (Robins, Elliott & Pattison, 2001).

Likewise, if the two classes comprise tie variables corresponding to two distinct measurement points, then a possible tie from *i* to *j* at time 2 might be assumed to be conditionally dependent on a possible tie from *k* to *i* at time 1 only if the tie from *k* to *i* is observed to be present at time 2 (i.e, only if it persists in time). Robins and Pattison (2001) refer to this as the *constant tie* assumption, and argue that it is plausible for some network ties.

Challenges

Many challenges remain. In statistical terms, the p^* class of models presents some important problems in a number of areas, including the estimation of parameters and their standard errors. Despite the promising steps that have been taken to obtain maximum likelihood parameter estimates for models in the p^* class, much work is required to further develop these techniques if they are to be applicable to models of the level of complexity that social theory is likely to demand. In addition, it would be useful to understand the dynamics of processes that have the models that we have constructed as their equilibrium distributions. The

Metropolis-Hastings algorithm provides one clue, as it defines a "birth-and-death" process that necessarily converges to the model used to define it; Snijders's (in press) model for network evolution provides another.

On the substantive side, the constructions that we have introduced suggest both empirical and theoretical tasks. We need a better understanding of the role of various neighborhood forms and setting structure hypotheses that are useful in different empirical contexts. An especially interesting version of this challenge is to develop an understanding of the interdependence of physical space and network forms, as indicated earlier. The more complex model forms generated by these new constructions also invite richer and more specific theoretical articulation. The work by Mische and Robins (2000) provides one compelling illustration of how this might be done, but the development of a fuller understanding of the interdependencies among social actors in dynamic, network-based settings (Abbott, 1997; Emirbayer, 1997; Emirbayer & Goodwin, 1994) poses an even greater challenge.

Appendix A. List of symbols

$N = \{1,2,...,n\}$	set of network nodes
x_{ij}	observed tie between nodes i and j ($x_{ij} = 1$ if present; $x_{ij} = 0$ otherwise)
$\mathbf{x} = [x_{ij}]$	$n \times n$ (binary) array $\mathbf{x}$ denoting an *observed* network on N:
X_{ij}	random variable for the *possible tie* (i, j) between nodes i and j
$\mathbf{X} = [X_{ij}]$	$n \times n$ (binary) array $\mathbf{X}$ denoting a *random* graph or network on N:
$\Pr(\mathbf{X} = \mathbf{x})$	probability that the random network $\mathbf{X}$ is equal to $\mathbf{x}$
$\mathbf{D}$	d*ependence graph* $\mathbf{D}$
M	node set of $\mathbf{D}$ (the set of all possible ties)
E	edge set of $\mathbf{D}$ (pairs of possible ties assumed conditionally dependent)
A	subset of possible ties in M
λ_A	model parameter corresponding to the subset A of possible ties
$z_A(\boldsymbol{x}) = \Pi_{(i,j)\in A}\, x_{ij}$	*network statistic* corresponding to subset A of M
$c = \Sigma_{\mathbf{X}} \exp\{\Sigma_A \lambda_A z_A\}$	normalising quantity
$[A]$	isomorphism class for a set A of possible ties
$z_{[A]}(\boldsymbol{x})$	count of number of configurations in $\boldsymbol{x}$ corresponding to class $[A]$
$s \subseteq M$	setting (subset of possible ties)
S	setting structure (a collection of settings)
H	set of all cliques of $\mathbf{D}$
$H_S = H \cap S$	clique-set of $\mathbf{D}$ restricted by the setting structure S
$\mathbf{D}_B$	*partial dependence structure*

Appendix B. Pseudolikelihood estimation of model parameters

Given an observed network and a proposed p^* model, it is naturally of interest to estimate the model parameters from the observed network. In view of the difficulty of maximum likelihood estimation of parameters (but see Corander, Dahmström & Dahmström, 1998; Crouch & Wasserman, 1998; Snijders, 2001; and also Ripley, 1988), Strauss and Ikeda (1990) suggested an alternative *pseudolikelihood* means of estimation. Following Besag (1977) and Strauss and Ikeda (1990), we define a *pseudolikelihood function*:

$$PL(\lambda) = \Pi_{i \neq j} \Pr(X_{ij}=1 \mid \boldsymbol{X}_{ij}^{c})^{x_{ij}} \Pr(X_{ij}=0 \mid \boldsymbol{X}_{ij}^{c})^{(1-x_{ij})}$$

where $\lambda = [\lambda_A]$ is a vector of parameters and $\boldsymbol{X}_{ij}^{c} = \{X_{kl}: k, l \in N, k \neq l, (k,l) \neq (i,j)\}$. The *maximum pseudo-likelihood estimator (MPLE)* is defined as the value of λ that maximizes $PL(\lambda)$. Strauss and Ikeda (1990) established that MPLEs for λ are equivalent to MLEs for λ assuming independent observations x_{ij} in the following (conditional) logit model (Strauss & Ikeda, 1990). Let $\boldsymbol{x}_{ij}^{+}$ denote the realisation $\boldsymbol{x}$ with x_{ij} set to 1 and let $\boldsymbol{x}_{ij}^{-}$ denote the realisation $\boldsymbol{x}$ with x_{ij} set to 0; recall also that $z_A(\boldsymbol{x})$ is the value of the statistic for the network $\boldsymbol{x}$ corresponding to the clique A of the dependence graph. Then

$$\begin{aligned} \text{logit} \Pr(X_{ij}=1 \mid \boldsymbol{X}_{ij}^{c}) &= \log[\Pr(X_{ij}=1 \mid \boldsymbol{X}_{ij}^{c}) / \Pr(X_{ij}=0 \mid \boldsymbol{X}_{ij}^{c})] \\ &= \Sigma_{A \subseteq M} \lambda_A [z_A(\boldsymbol{x}_{ij}^{+}) - z_A(\boldsymbol{x}_{ij}^{-})] \\ &= \Sigma_{A \subseteq M} \lambda_A (d_A)_{ij} \end{aligned}$$

where $(d_A)_{ij} = [z_A(\boldsymbol{x}_{ij}^{+}) - z_A(\boldsymbol{x}_{ij}^{-})]$ is the *change* in the value of the statistic $z_A(\boldsymbol{x})$ when x_{ij} changes from 1 to 0. Thus, MPLEs can be obtained from standard logistic regression programs (with the observed x_{ij}'s as the response variable, and the $(d_P)_{ij}$'s as explanatory variables).

Let $\boldsymbol{d}_A = [(d_A)_{ij}]$ be the *change statistic matrix* corresponding to the clique A of $\mathbf{D}$. The matrix of change statistics needs to be computed for each clique A of $\mathbf{D}$. In table A1, we give matrix expressions for $\boldsymbol{d}_A$ in the case of homogeneous Markov models for nondirected and directed networks. Table A2 provides expressions for the change statistic matrices used for the setting-restricted and non-markovian models described in Section 5.

We note that in order to obtain MPLEs for model parameters, it is sufficient to fit a logistic regression model in which x_{ij} serves as the observation on a response variable, and $(d_A)_{ij}$ constitute explanatory variables. (The cases for the analysis are indexed by M.) The coefficient for $(d_A)_{ij}$ is the PLE for λ_A. Note that the number of explanatory variables is equal to the number of classes $[A]$ of cliques whose parameters are to be estimated.

Appendix C. Proof of proposition

Following Besag (1974) we define $Q(\boldsymbol{x}) = \ln\{\Pr(\boldsymbol{X} = \boldsymbol{x})/\Pr(\boldsymbol{X} = \mathbf{0})\}$, where $\mathbf{0}$ is the null graph; we also define $\boldsymbol{x}_{ij}^{-}$ according to $(x_{ij}^{-})_{kl} = (x_{ij})_{kl}$ if $(k,l) \neq (i,j)$ and $(x_{ij}^{-})_{ij} = 0$, that is, $\boldsymbol{x}_{ij}^{-}$ is the same as $\boldsymbol{x}$ but its (i,j) entry is set to 0. Likewise, $\boldsymbol{x}_{ij}^{c}$ is defined according to $(x_{ij}^{c})_{kl} = (x_{ij})_{kl}$ if $(k,l) \neq (i,j)$ and $(x_{ij}^{c})_{ij}$ is regarded as missing; thus, $\boldsymbol{x}_{ij}^{c}$ is the same as $\boldsymbol{x}$ but its (i,j) entry is excluded.

Then

$$Q(\boldsymbol{x}) - Q(\boldsymbol{x}_{ij}^{-}) = \ln [\Pr(\boldsymbol{X} = \boldsymbol{x})/\Pr(\boldsymbol{X} = \boldsymbol{x}_{ij}^{-})]$$

$$= \ln [\Pr(X_{ij} = x_{ij} \mid \boldsymbol{X}_{ij}^{c} = \boldsymbol{x}_{ij}^{c})/\Pr(X_{ij} = 0 \mid \boldsymbol{X}_{ij}^{c} = \boldsymbol{x}_{ij}^{c})]$$

It follows from the Hammersley-Clifford theorem that

$$Q(\boldsymbol{x}) - Q(\boldsymbol{x}_{ij}^{-}) = x_{ij} \{\Sigma_{A \subseteq M \setminus \{(i,j)\}} \lambda_{A \cup \{(i,j)\}} \Pi_{(k,l) \in A}\, x_{kl}\}$$

where $\lambda_{A \cup \{(i,j)\}} = 0$ unless $A \cup \{(i,j)\}$ is a clique of $\mathbf{D}$.

Now suppose that C is not a clique of $\mathbf{D}_B$; that is, suppose without loss of generality that $((i,j),(k,l))$ is not an edge in $\mathbf{D}_B$ for some (k,l) with $(i,j), (k,l) \in C \subseteq M \setminus B$.

Consider the function $Q(\boldsymbol{x}) - Q(\boldsymbol{x}_{ij}^{-})$ for the case where $X_{mh} = 0$ for all $(m,h) \in B$.

Since X_{ij} is conditionally independent of X_{kl} in this case, $Q(\boldsymbol{x}) - Q(\boldsymbol{x}_{ij}^{-})$ must be independent of X_{kl} for all $\boldsymbol{x}_{ij}^{c}$ having $x_{mh} = 0$ for all $(m,h) \in B$. It therefore follows that $\lambda_{\{(i,j),(k,l)\}} = 0$ (by choosing $X_{mh} = 0$ for all $(m,h) \neq (i,j)$ or (k,l)).

Likewise, if $(i,j), (k,l), (p,q) \in C \subseteq M \setminus B$, we can choose $X_{mh} = 0$ for all $(m,h) \neq (i,j)$ or (k,l) or (p,q), from which it follows that

$$Q(\boldsymbol{x}) - Q(\boldsymbol{x}_{ij}^{-}) = \lambda_{\{(i,j),(k,l),(p,q)\}}\, x_{ij}x_{kl}x_{pq} = 0, \text{ and so } \lambda_{\{(i,j),(k,l),(p,q)\}} = 0.$$

A similar argument holds for all other subsets of C including (i,j) and (k,l), and so the result holds.

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Table 1. Fit of models to friendship ties in the Bank Wiring Room

Model	no. of parameters	-2LPL	MAR	parameter estimates edge	2-star	triangle
Universal setting structure						
Bernoulli	1	74.6	0.245	-1.79 (.30)	-	-
2-parameter Markov	2	71.1	0.238	-3.00 (.79)	.318 (.177)	-
3-parameter Markov	3	38.4	0 .112	-2.66 (.93)	-.294 (.278)	3.19 (.805)
Organizational setting structure						
2-parameter Markov	2	52.1	0.176	-4.57 (1.1)	1.09 (.320)	
3-parameter Markov	3	35.3	0.103	-3.40 (.81)	.023 (.416)	3.02 (1.04)

Table 2. Fit of models to reciprocated Work ties in the law firm (Lazega, 1993)

Model	no. of parameters	-2LPL	MAR
1. Bernoulli model	1	2119.0	0.258
2. level 3 Markov model	4	1760.8	0.213
3. level 4 3-semi-path model	7	1579.2	0.191
4. reduced 3-semi-path model	6	1598.9	0.193

Table 3. Parameter estimates for Markov and reduced 3-path models for reciprocated Work ties

model	configuration	PLE	approx s.e.
level 3 Markov			
	edge	-2.785	.369
	2-star	-0.019	.030
	3-star	0.002	.002
	3-cycle	0.482	.035
reduced 3-path model			
	edge	-3.669	.474
	2-star	0.307	.053
	3-star	-0.001	.002
	3-cycle	0.173	.047
	3-path	-0.019	.002
	4-cycle	0.086	.009

Table 4: Some classes of dependence assumptions

ties are conditionally dependent within:

1-paths	Bernoulli models
2-cycles	dyad-independent models
2-paths	2-path models
2-semi-paths	Markov models
3-paths	3-path models
3-semi-paths	3-semi-path models
…	
settings	setting-restricted versions of the above models

Table A. Computation of change statistics

Configuration corresponding to A	Form of A	$\boldsymbol{d}_A$
Homogeneous Markov model for a nondirected graph		
edge	$\{(i,j)\}$	$\boldsymbol{u}$
2-star	$\{(i,j),(i,k)\}$	$\boldsymbol{ux} + \boldsymbol{xu}$
3-star	$\{(i,j),(i,k),(i,l)\}$	$\boldsymbol{ux} * (\boldsymbol{ux} - 1)/2 + \boldsymbol{xu} * (\boldsymbol{xu} - 1)/2$
…		
triangle	$\{(i,j),(i,k),(j,k)\}$	$\boldsymbol{xx}$
Additional parameters in Example 1 (overlapping subgroup setting structure for a non-directed graph)		
2-star	$\{(i,j),(j,k)\}$	$\{\Sigma_{1\leq k\leq K} (-1)^{k-1} \Sigma_{Q:\lvert Q\rvert=k} [\boldsymbol{N}_Q(\boldsymbol{x}*\boldsymbol{N}_Q) + (\boldsymbol{x}*\boldsymbol{N}_Q)\boldsymbol{N}_Q]\}_{ij}$
3-cycle	$\{(i,j),(j,k),(k,i)\}$	$\{\Sigma_{1\leq k\leq K} (-1)^{k-1} \Sigma_{Q:\lvert Q\rvert=k} [(\boldsymbol{x}*\boldsymbol{N}_Q)(\boldsymbol{x}*\boldsymbol{N}_Q)]\}_{ij}$
Additional parameters for examples 2 and 3 (3-paths & 4-cycles in a non-directed graph)		
3-path	$\{(i,j),(j,k),(k,l)\}$	$(\boldsymbol{uxx})_{ij} - (\boldsymbol{xx})_{jj} - \boldsymbol{x}_{ij}(\boldsymbol{ux})_{ii} + \boldsymbol{x}_{ij}$ $+ (\boldsymbol{xux})_{ij} - \boldsymbol{x}_{ij}(\boldsymbol{ux})_{jj} - \boldsymbol{x}_{ij}(\boldsymbol{xu})_{ii} + \boldsymbol{x}_{ij}$ $+ (\boldsymbol{xxu})_{ij} - \boldsymbol{x}_{ij}(\boldsymbol{xu})_{jj} - (\boldsymbol{xx})_{ii} + \boldsymbol{x}_{ij}$
4-cycle	$\{(i,j),(j,k),(k,l),(l,i)\}$	$(\boldsymbol{xxx})_{ij} - \boldsymbol{x}_{ij}(\boldsymbol{xx})_{jj} - \boldsymbol{x}_{ij}(\boldsymbol{xx})_{ii} + \boldsymbol{x}_{ij}$

where:

(1) $u_{ij} = 1$ if X_{ij} is a possible tie, and $u_{ij} = 0$, otherwise;

(2) for matrices $\boldsymbol{a}$ and $\boldsymbol{b}$ and constant k:
$(\boldsymbol{a}*\boldsymbol{b}) = [a_{ij}b_{ij}]$ (element-wise multiplication)
$\boldsymbol{a} + k = [a_{ij} + k]$;

(3) Markov configurations are shown in Figure 1

(4) for example 1, there are K maximal subgroups, indexed by the set $P = \{1,2,\ldots,K\}$;

(5) $(n_q)_{ij} = 1$ if X_{ij} is a possible tie in the qth maximal subgroup and $(n_q)_{ij} = 0$, otherwise;

(6) $(N_Q)_{ij} = 1$ if $(n_q)_{ij} = 1$ for all $q \in Q \subseteq P$; and

(7) for example 3, $\boldsymbol{x}$ is regarded as a symmetric matrix on the union of the set of women and the set of events.

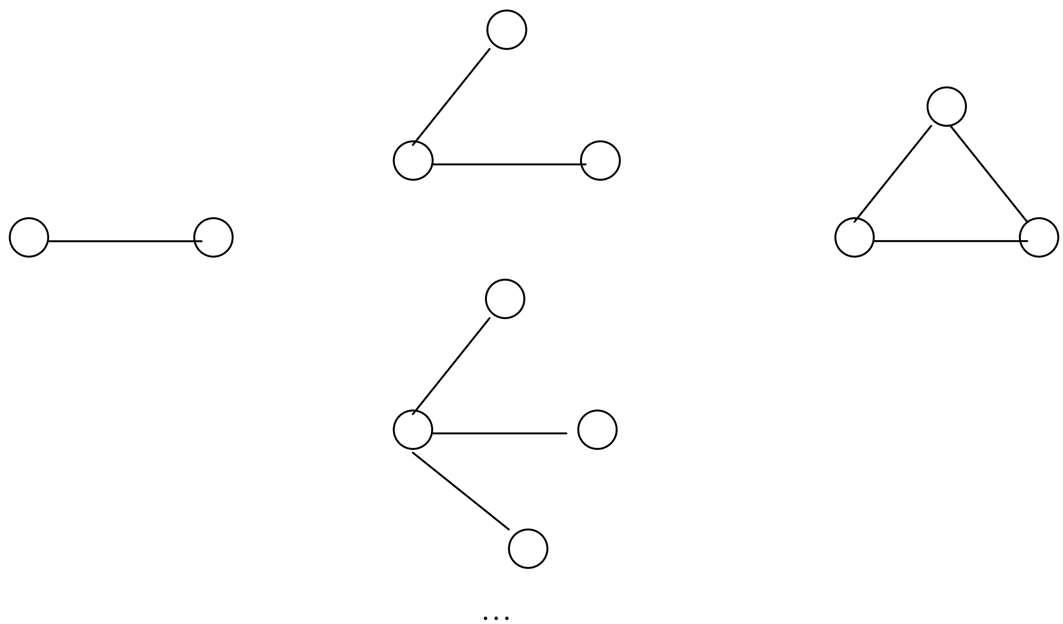

Figure 1. Configurations with three or fewer edges corresponding to Markovian neighborhoods for a graph

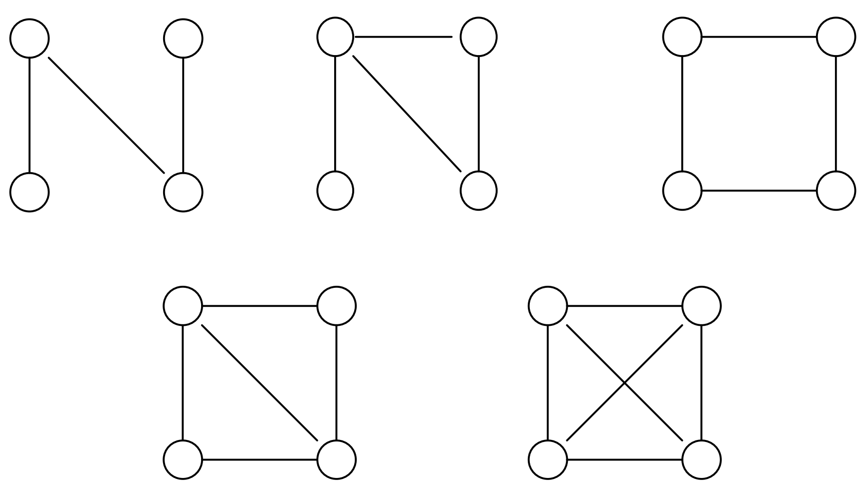

Figure 2. Non-Markovian configurations corresponding to non-zero parameters in a 3-path random graph model