

## Cohesive subsets of nodes:

(Wasserman & Faust, 1994, chapter 7)

- Theoretical background
- Review terminology
- Cliques
- Subgroups based on reachability:
  - n-cliques
  - n-clans
- Subgroups based on nodal degree
  - k-plexes
  - k-cores

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## A short theoretical background

- Notions of social group widely used in social sciences.
- But where do groups come from?
- An early network-based definition of a group:
  - “A group is defined by the interactions of its members. If we say that individuals A, B, C, D, E . . . form a group, this will mean that at least the following circumstances hold. Within a given period of time, A interacts more often with B, C, D, E . . . than he does with M, N, L, O, P . . . whom we choose to consider outsiders or members of other groups. .... It is possible just by counting interactions to map out a group quantitatively distinct from others.” (Homans, 1950, p.84).

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## Review some graph-theoretic terminology

- A graph is a set of nodes and a set of edges between them.
  - Two nodes are *adjacent* if there is an edge between them. Two edges are *adjacent* if they share a node.
  - A *subgraph* is a subset of the nodes and the set of edges among them from the original graph.
  - A subgraph  $S$  is *maximal* with respect to a property if the property is true for  $S$  but not true for any other subgraph that contains  $S$ .
- A *complete* graph (subgraph) has all possible edges present.
- A *path* is a sequence of adjacent edges connecting two nodes; if there is a path between two nodes they are *reachable*. A shortest path between two nodes is a *geodesic*. A graph is *connected* if all pairs of nodes are reachable. The *length* of a path is the number of edges in it.
- The *degree* of a node is the number of nodes adjacent to it.
- A *component* is a maximal connected subgraph.

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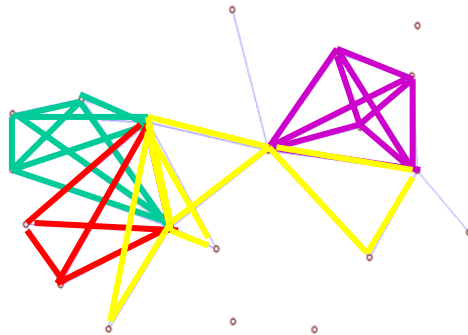
## Cliques

- A clique in a graph is a maximal complete subgraph of three or more nodes.
  - All nodes in the clique are adjacent to each other. No other nodes are adjacent to all nodes in the clique.
- Notice that cliques in a graph may overlap.
- A clique is a very stringent definition of a cohesive subgroup

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## Cohesive subgroups of actors

### Cliques



Friendship network  
(Robins, 2002)

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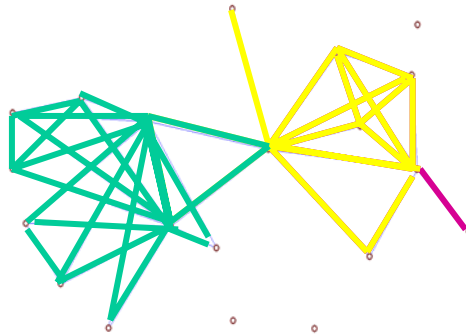
## Subgroups based on reachability: *n*-clique

- An *n*-clique is a maximal subgraph in which the largest geodesic distance between any two nodes is no greater than *n*.
  - A 2-clique is based on a subset of nodes where there is a path of length 2 (or less) between any pair of nodes.
  - A 1-clique is just a clique.

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### 2-Cliques



Friendship network  
(Robins, 2002)

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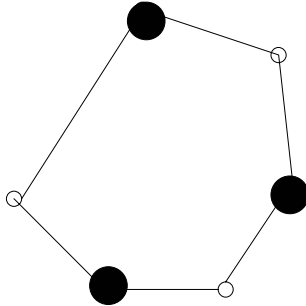
## Subgroups based on reachability: *n*-clan

- An *n*-clique can be defined on the basis of geodesics that pass through nodes not in the *n*-clique. The *n*-clique may not even be connected.
- An *n*-clan is an *n*-clique based on paths that occur only on nodes within the *n*-clique.

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**Cohesive subgroups of actors**

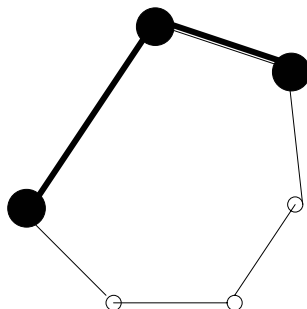
**A disconnected 2-Clique**



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**Cohesive subgroups of actors**

**A 2-Clan**



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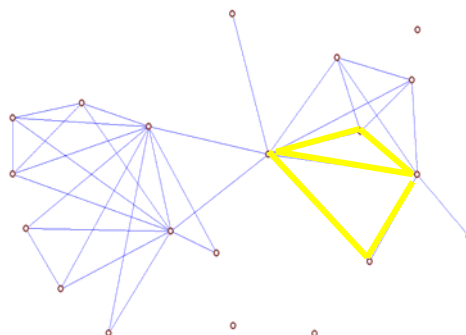
## Subgroups based on degree: *k*-plex

- A clique occurs when all nodes in a subgroup are adjacent.
- Subgroups based on degree are determined such that all nodes be adjacent to a *certain number* of other nodes in the group.
- A *k*-plex is a maximal subgraph of  $g$  nodes where each node is adjacent to at least  $g - k$  nodes in the subgraph.
  - A 2-plex requires that all nodes be adjacent to  $g - 2$  nodes.
  - A 1-plex, where all nodes are adjacent to  $g - 1$  nodes, is just a clique.

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#### A 2-plex



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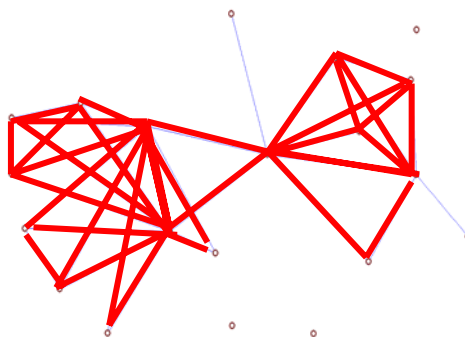
## Subgroups based on degree: *k*-core

- A *k*-core is a maximal subgraph where each node is adjacent to at least *k* nodes in the subgraph.
  - A *k*-core is defined in terms of the minimum degree within the subgraph.

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### Cohesive subgroups of actors

#### 2-core

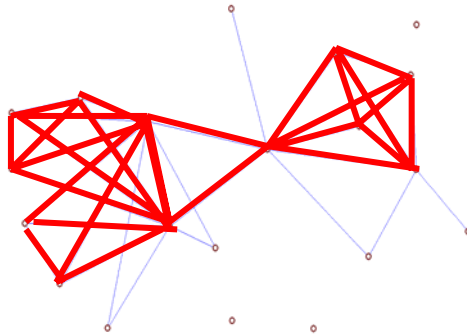


Friendship network  
(Robins, 2002)

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## Cohesive subgroups of actors

### 3-core

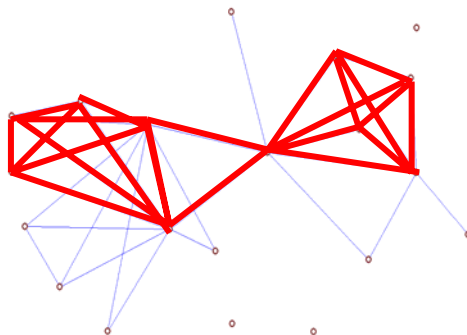


Friendship network  
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## Cohesive subgroups of actors

### 4-core



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